MATHEMATICS-X

MODULE-2

INDEX

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AREA RELATED TO CIRCLES

INTRODUCTION

In earlier classes, we have studied methods of finding perimeters and areas of simple plane figures such as rectangles, squares, parallelograms, triangles and circles. In our daily life, we come across many objects which are related to circular shape in some form or the other. For example, cycle wheels, wheel arrow, drain cover, bangles, flower beds, circular paths etc. That is why the problem of finding perimeters and areas related to circular figures is of great practical importance. In this chapter, we shall discuss problems on finding the areas of some combinations of plane figures involving circles or parts of circles. Let us first recall the concepts related to their perimeter and area of a circle.

★ HISTORICAL FACTS

Mensuration is that branch of mathematics which studies the method of measurements. Measurement is a very important human activity. We measure the length of a cloth for stitching. The area of a wall for painting, the perimeter of a plot for fencing. We do many other measurements of similar nature in our daily life. All these measurements, we shall study in this chapter called Mensuration.

π(pi) occupies the most significant place in measurement of surface area as well as volume of various solid and plane figures. The value of π is not exactly known. The story of the accuracy by which the value of π was estimated is an interesting one.

ARYABHATA

Mathematically $\pi = \dfrac{\text{Circumference of a circle}}{\text{Diameter of the circle}}$

$$\pi = \frac{(100+4)\times 8+62000}{20,000} = \frac{62832}{20,000} = 3.1416$$

S. RAMANUJAN

According to **S. Ramanujan,** the value of π

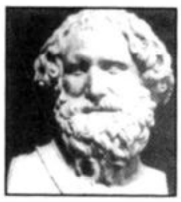
$$\pi = \frac{355}{113}$$

PTOLEMY

According to **Archimedes** ; the value of π is given below :

$$\pi = \frac{\text{Circumference of a circle}}{\text{Diameter of the circle}} = 3\frac{10}{71} < \pi < 3\frac{10}{70}$$

Ptolemy, $\pi = 3\dfrac{17}{120}$



ARCHIMEDES

The **Egyptians,** $\pi = \left(\dfrac{16}{9}\right)^2 = 3.160$

Note : π (pi) is an irrational number. It cannot be expressed as the ratio of whole numbers. However, the ratio 22 : 7 is often used as approximation for it.

Basic concepts and important results

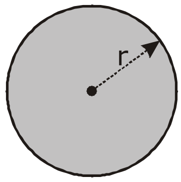
* **Perimeter and area of a circle**

The perimeter of circle is the length of its boundary. The unit of measurement of perimeter is the unit of length.

The area of a circle is the measurement of the region enclosed by its boundary.

It is measured in square units i.e., square centimetres or square metres etc.

* **Circumference and area of a circle**



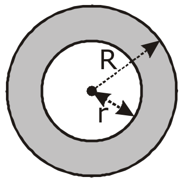
If r is the radius of a circle, then

(i) the circumference of the circle = 2π r.

(ii) the area of the circle = π r^2.

By the area of a circle we mean the area of a circular region.

* **Area of a circular ring**

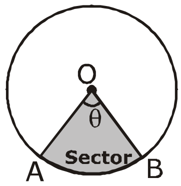


If R and r are the radii of the bigger and smaller (concentric) circles, then

are of ring (shaded portion) = $\pi(R^2 - r^2)$.

* **Perimeter and area of a sector of a circle**

Let OAB (shown shaded) be a sector of a circle with centre O and radius r. Let θ be the degree measure of $\angle AOB$, then



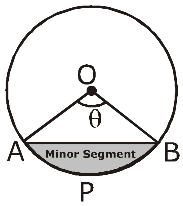
(i) the length of the arc of sector = $\frac{\theta}{360} \times 2\pi r$

(ii) the perimeter of the sector = $\frac{\theta}{360} \times 2\pi r + 2r$

(iii) the area of the sector = $\frac{\theta}{360} \times \pi r^2$

* **Perimeter and area of a segment of a circle**

Let APB (shown shaded) be a segment of a circle with centre O and radius r. Let θ be the degree measure of $\angle AOB$, then



(i) the perimeter of segment = $\frac{\theta}{360}$ × 2π r + length of AB

(ii) the area of the segment APB = area of sector OAB − area of ΔOAB

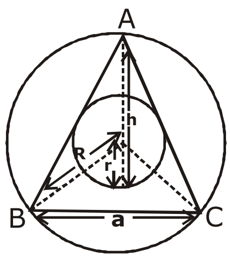
Note : When we write 'sector' and 'segment' of a circle we will mean 'minor sector' and 'minor segment' unless stated otherwise.

* **Circumference and area of circumscribed and inscribed circles of an equilateral triangle.**

Let ABC be an equilateral triangle of side a and height h, then h = $\frac{\sqrt{3}}{2}$a.

If R and r are the radii of the circumscribed and inscribed circles of ΔABC, then

R = $\frac{2}{3}$h and r = $\frac{1}{3}$h, and



(i) the circumference of the circumscribed circle = $2\pi R^2 = \frac{4}{3}\pi h$

(ii) the area of the circumscribed circle = $\pi R^2 = \frac{4}{9}\pi h^2$

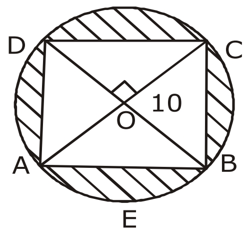
(iii) the circumference of the inscribed circle = $2\pi r = \frac{2}{3}\pi h$

(iv) the area of the inscribed circle = $\pi r^2 = \frac{1}{9}\pi h^2$.

SOLVED PROBLEMS

Ex.1 A square ABCD is inscribed in a circle of radius 10 units. Find the area of the circle, not included in the square. (Use $\pi = 3.14$)

Sol. Area of the segment AEB = Area of sector OAEB – Area of ΔAOB



Area of sector OAEB

$= \frac{\pi r^2 \theta}{360^\circ} = \frac{3.14 \times (10)^2 \times 90^\circ}{360^\circ}$

$= \frac{3.14 \times 100 \times 90^\circ}{360^\circ} = 78.5$ sq. units

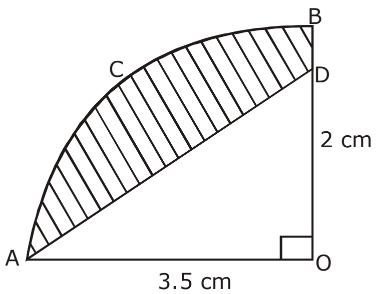
ar(ΔOAB) = $\frac{1}{2} r^2 \sin\theta = \frac{1}{2} \times (10)^2 \sin 90^\circ$

$= \frac{1}{2} \times 100 \times 1$ sq. units = 50 sq. units. From (i),

$\therefore$ Area of the segment AEB = 78.5 – 50 = 28.5 sq. units.

$\therefore$ Area of the circle, not included in the square
= 4 × Area of the segment AEB
= 4 × 28.5 sq. units = 114.0 sq. units = 114 sq. units.

Ex.2 In the figure OACB represents a quadrant of circle of radius 3.5 cm with centre O. **[NCERT]**
(i) Calculate the area of quadrant OACB.
(ii) Given OD = 2 cm. Calculate the area of shaded portion.
$\left(\text{Take } \pi = \frac{22}{7}\right)$.



Sol. Area of quadrant = $\frac{1}{4}$ area of the circle

$= \frac{1}{4}\pi r^2 = \frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5$

$= \frac{77}{8}$ sq. cm = 9.625 sq. cm

Area of rt. ΔAOD = $\frac{1}{2} \times OA \times OD$

$= \frac{1}{2} \times 3.5 \times 2 = 3.5 \text{ cm}^2$

$\therefore$ Area of the shaded region
= Area of the quadrant – Area of ΔAOD
= 9.625 – 3.5 = 6.125 cm^2.

Ex.3 The minute hand of a clock is 10 cm long. Find the area swept by the minute hand between 9:00 a.m. and 9:35 a.m.

Sol. The minute hand describes a circle of radius 10 cm. In an hour, i.e, 60 minutes it describes an angle of 360º.

So, in 35 minutes, the minute hand describes an angle of

$$= \frac{360^\circ \times 35}{60} = 210^\circ$$

Thus, the minute hand describes a sector of angle 210º and radius 10 cm in 35 minutes.

Required area swept by the minute hand

= Area of the sector

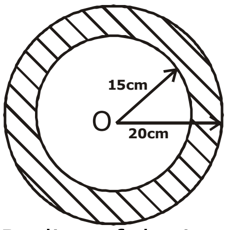
$$= \frac{\pi r^2\theta}{360^\circ} = \frac{22\times(10)^2 210}{7\times 360^\circ}$$

$$= \frac{22\times 10\times 10210}{7\times 360^\circ} = \frac{500}{3}$$

$= 183.33\ cm^2$

Ex.4 Find the area of a ring -shaped region enclosed between two concentric circles of radii 20 cm and 15 cm.

Sol.



Radius of the inner circle (r_1) = 15 cm

Radius of the outer circle (r_2) = 20 cm

Area of the ring shaped region

= Area of the outer circle – Area of the inner circle

$$= \pi r_2^2 - \pi r_1^2 = \pi(20)^2 - \pi(15)^2$$

$= \pi\ [(20^2 - (15)^2]$

$$= \frac{22}{7} \times (20+15)\ (20-15)$$

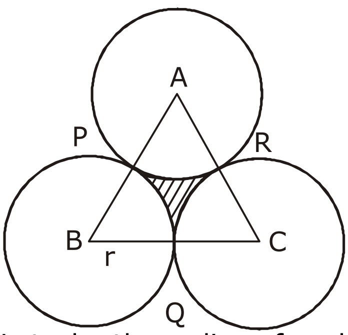
$[\because\ a^2 - b^2 = (a + b)\ (a - b)]$

$$= \frac{22}{7} \times 35 \times 5 = 550\ cm^2.$$

Ex.5 The area of an equilateral triangle is 17320.5 cm^2. With each vertex as centre, a circle is described with radius equal to half the length of the side of the traingle. Find the area of the shaded region

(Use $\pi = 3.14$ and $\sqrt{3} = 1.73205$) **[NCERT]**

Sol.



Let r be the radius of each circle and a is the length of each side of the equilateral triangle.

Area of equilateral triangle = $\frac{\sqrt{3}}{4}a^2$

$\therefore\quad 17320.5 = \frac{\sqrt{3}}{4}a^2$

$$\frac{17320.5 \times 4}{\sqrt{3}} = a^2$$

or $a^2 = \dfrac{17320.5 \times 4}{1.73205}$

$a^2 = \dfrac{17320.5 \times 4 \times 10000}{173205} = 40000$

a = 200 cm.

$\therefore$ Length of each side of the equilateral triangle = 200 cm

Radius of each circle (r) = $\dfrac{a}{2} = \dfrac{200}{2}$

= 100 cm

Area of the sector PAR in circle

$= \dfrac{\pi r^2 \theta}{360} = \dfrac{3.14 \times (100)^2 \times 60}{360}$

$= \dfrac{314}{100} \times \dfrac{100 \times 100 \times 60}{360} = \dfrac{15700}{3}$ cm^2

Total area of 3 sectors

$= 3 \times \dfrac{15700}{3} = 15700$ cm^2

$\therefore$ Area of the shaded region

= Area of the equilateral triangle – Area of 3 equal sectors

= (17320.5 – 15700) cm^2 = 1620.5 cm^2.

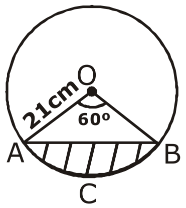
Ex.6 In a circle of radius 21 cm. An arc subtends an angle of 60º at the centre. Find

(i) the length of the arc

(ii) area of sector formed by the arc

(iii) The area of the segment made by this arc.

Sol.



(i) Length of the arc

$= \dfrac{\pi r \theta}{180} = \dfrac{22}{7} \times 21 \times \dfrac{60}{180} = 22$ cm.

(ii) Area of the sector OACB = $\dfrac{\pi r^2 \theta}{360}$

$= \dfrac{22}{7} \times (21)^2 \times \dfrac{60}{180} = \dfrac{22}{7} \times 21 \times 21 \times \dfrac{60}{180} = 231$ cm^2.

(iii) Area of the segment ACB = Area of the sector OACB

– Area of the equilateral triangle OAB ....(1)

Area of equilateral ΔOAB = $\dfrac{\sqrt{3}}{4}$ (side)^2

$= \dfrac{\sqrt{3}}{4} \times 21 \times 21 = \dfrac{1.73 \times 441}{4} = \dfrac{762 - 93}{4} = 190.73$ cm^2.

From (1), Area of the segment ACB = (231 – 190.73) cm^2 = 40.27 cm^2.

Take $\pi = \dfrac{\mathbf{22}}{\mathbf{7}}$, unless stated otherwise

Ex.7 Calculate the circumference and area of a circle of radius 5·6 cm.

Sol. We have :

Circumference of the circle = $2\pi r = \left(2 \times \frac{22}{7} \times 5{\cdot}6\right)$ cm = 35.2 cm.

Area of the circle = $\pi r^2 = \left(\frac{22}{7} \times 5{\cdot}6 \times 5{\cdot}6\right)$ cm^2 = 98· 56 cm^2 .

Ex.8 The circumference of a circle is 123·2 cm. Calculate :

(i) the radius of the circle in cm,

(ii) the area of the circle, correct to nearest cm^2.

Sol. (i) Let the radius of the circle be r cm.

Then, its circumference = $(2\pi r)$ cm.

$\therefore$ $2\pi r = 123{\cdot}2$ $\Rightarrow 2 \times \frac{22}{7} \times r = 123{\cdot}2$

$\Rightarrow$ $r = (123.2 \times \frac{7}{44}) = 19{\cdot}6$ cm.

$\therefore$ Radius of the circle = 19·6 cm.

(ii) Area of the circle = $\pi r^2 = (\frac{22}{7} \times 19{\cdot}6 \times 19.6)$ cm^2 = 1207·36 cm^2 .

$\therefore$ Area of the circle, correct to nearest cm^2 = 1207 cm^2.

Ex.9 The area of a circle is 301·84 cm^2. Calculate:

(i) the radius of the circle in cm.

(ii) the circumference of the circle, correct to nearest cm.

Sol. (i) Let the radius of the circle be r cm.

Then, its area = πr^2 cm^2 = 301.84

$\Rightarrow$ $\frac{22}{7} \times r^2 = 301.84$

$\Rightarrow$ $r^2 = \left(301.84 \times \frac{7}{22}\right) = 96.04$

$\Rightarrow$ $r = \sqrt{96.04} = 9{\cdot}8$ cm.

$\therefore$ Radius of the circle = 9·8 cm.

(ii) Circumference of the circle

$= 2\pi r = \left(2 \times \frac{22}{7} \times 9.8\right)$ cm = 61.6 cm.

$\therefore$ Circumference of the circle, correct to nearest cm = 62 cm.

Ex.10 The perimeter of a semi-circular protractor is 32·4 cm. Calculate :

(i) the radius of the protractor in cm,

(ii) the area of the protractor in cm^2.

Sol. (i) Let the radius of the protractor be r cm.

Then, its perimeter = $(\pi r + 2r)$ cm.

$\therefore$ $\pi r + 2r = 32{\cdot}4$ $\Rightarrow (\pi + 2)r = 32{\cdot}4$

$\Rightarrow$ $\left(\frac{22}{7} + 2\right) r = 32.4$ $\Rightarrow \frac{36}{7} r = 32.4$

$\Rightarrow$ $r = \left(32.4 \times \frac{7}{36}\right)$ cm = 6.3 cm.

Radius of the protractor = 6.3 cm.

(ii) Area of the protractor = $\frac{1}{2}\pi r^2$

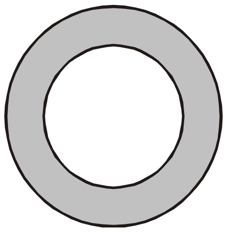
$= \left(\frac{1}{2} \times \frac{22}{7} \times 6.3 \times 6.3\right)$ cm^2 = 62·37cm^2.

$\therefore$ Area of the protractor = 62·37 cm^2.

Ex.11 The area enclosed by the circumferences of two concentric circles is 346.5 cm^2. If the circumference of the inner circle is 88 cm, calculate the radius of the outer circle.

Sol. Let the radius of inner circle be r cm.

Then, its circumference = $(2\pi r)$ cm.



$\therefore\ 2\pi r = 88 \Rightarrow 2 \times \frac{22}{7} \times r = 88$

$\Rightarrow r = \left(88 \times \frac{7}{44}\right) = 14$ cm.

$\therefore$ Radius of the inner circle is, r = 14 cm.

Let the radius of the outer circle be R cm.

Then, area of the ring = $(\pi R^2 - \pi r^2)\ cm^2$

$= \pi(R^2 - r^2)cm^2$

$= \frac{22}{7} \times [R^2 - (14)^2]\ cm^2$

$= \left(\frac{22}{7}R^2 - 616\right) cm^2$

$\therefore\ \frac{22}{7}R^2 - 616 = 346.5 \Rightarrow \frac{22}{7}R^2 = 962.5$

$\Rightarrow\ R^2 = \left(962.5 \times \frac{7}{22}\right) = 306.25$

$\Rightarrow\ R = \sqrt{306.25} = 17{\cdot}5$ cm.

Hence, the radius of the outer circle is 17·5 cm.

Ex.12 Two circles touch externally. The sum of their areas is 130π sq. cm and the distance between their centres is 14 cm. Determine the radii of the circles.

Sol. Let the radii of the given circles be R cm and r cm respectively. As the circles touch externally, distance between their centres = (R + r) cm.

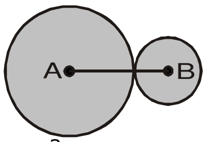
$\therefore\ R + r = 14$ ...(i)

Sum of their areas = $(\pi R^2 + \pi r^2)cm^2 = \pi(R^2 + r^2)\ cm^2$.

$\therefore\ \pi(R^2 + r^2) = 130\pi$

$\Rightarrow\ R^2 + r^2 = 130$ ...(ii)

We have the identity, $(R+r)^2 + (R-r)^2 = 2(R^2+r^2)$



$(14)^2 + (R - r)^2 = 2 \times 130$ [From (i) and (ii)]

$(R - r)^2 = 64$

$R - r = 8$ ... (iii)

On solving (i) and (iii), we get R = 11 and r = 3.

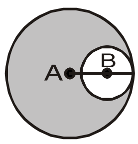
Hence, the radii of the given circles are 11 cm and 3 cm.

Ex.13 Two circles touch internally. The sum of their areas is 116π sq. cm and the distance between their centres is 6 cm. Find the radii of the given circles.

Sol. Let the radii of the given circles be R cm and r cm respectively. As the circles touch internally, distance between their centres = (R – r) cm.

$\therefore\ R - r = 6$ ... (i)

Sum of theIr areas



$= (\pi R^2 + \pi r^2)cm^2 = \pi(R^2 + r^2)\ cm^2$

$\therefore\ \pi(R^2 + r^2) = 116\ \pi$

$\Rightarrow\ R^2 + r^2 = 116$ ....(ii)

We have the identity, $(R+r)^2 + (R-r)^2 = 2(R^2+r^2)$

$\Rightarrow\ (R + r)^2 + 6^2 = 2 \times 116$ [Using (i) and (ii)]

$\Rightarrow\ (R + r)^2 = 196$

$\Rightarrow\ R + r = \sqrt{196} = 14$ ...(iii)

On solving (i) and (iii), we get R = 10 and r=4.

Hence, the radii of the given circles are 10 cm and 4 cm .

Ex.14 The wheel of a cart is making 5 revolutions per second. If the diameter of the wheel is 84 cm, find its speed in km/hr. Give your answer, correct to nearest km.

Sol. Radius of the wheel = 42 cm.

Circumference of the wheel = $2\pi r = \left(2 \times \frac{22}{7} \times 42\right)$ cm = 264 cm .

Distance moved by the wheel in 1 revolution = 264 cm.

Distance moved by the wheel in 5 revolutions = (264 × 5) cm = 1320 cm.

∴ Distance moved by the wheel in 1 second = 1320 cm.

Distance moved by the wheel in 1 hour = (1320 × 60 × 60) cm.

$= \left(\frac{1320 \times 60 \times 60}{100 \times 1000}\right)$ km

∴ Speed of the cart = $\left(\frac{1320 \times 60 \times 60}{100 \times 1000}\right)$ km/hr = 47·52 km/hr.

Hence, the speed of the cart, correct to nearest km/hr is 48 km/hr.

Ex.15 The diameter of the driving wheel of a bus is 140 cm. How many revolutions must the wheel make in order to keep a speed of 66 km/hr?

Sol. Distance to be covered in 1 min. = $\left(\frac{66 \times 1000}{60}\right)$ m = 1100 m.

Radius of the wheel = $\left(\frac{140}{2}\right)$ cm

= 70 cm = 0.70 m.

Circumference of the wheel

$= 2\pi r = \left(2 \times \frac{22}{7} \times 0.70\right)$ m = 4.4 m.

∴ Number of revolutions per minute

$= \left(\frac{1100}{4.4}\right) = 250.$

Hence, the wheel must make 250 revolutions per minute.

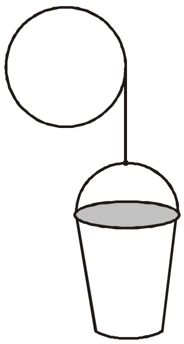
Ex.16 A bucket is raised from a well by means of a rope which is wound round a wheel of diameter 77 cm. Given that the bucket ascends in 1 min. 28 seconds with a uniform speed of 1·1 m / sec, calculate the number of complete revolutions the wheel makes in raising the bucket.

Sol. Time taken by bucket to ascend = 1 min. 28 sec. = 88 sec. Speed = 1.1 m/sec.

Length of the rope = Distance covered by bucket to ascend

= (1·1×88)m=(1·1×88×100)cm = 9680 cm.

Radius of the wheel = $\frac{77}{2}$ cm.



Circumference of the wheel

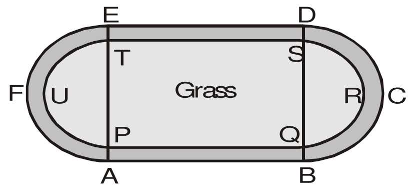
$= 2\pi r = \left(2 \times \frac{22}{7} \times \frac{77}{2}\right)$ cm = 242 cm.

∴ Number of revolutions

$= \frac{\text{Length of the rope}}{\text{Circumference of the wheel}} = \left(\frac{9680}{242}\right) = 40.$

Hence, the wheel makes 40 revolutions to raise the bucket.

Ex.17 The figure shows a running track surrounding a grass enclosure PQRSTU. The enclosure consists of a rectangle PQST with a semi-circular region at each end. Given, PQ = 200 m and PT = 70 m.



(i) Calculate the area of the grassed enclosure in m^2.

(ii) Given that the track is of constant width 7 m, calculate the outer perimeter ABCDEF of the track.

Sol. (i) Diameter of each semi-circular region of grassed enclosure = PT = 70 m,

$\therefore$ Radius of each one of them = 35 m.

Area of grassed enclosure

= (Area of rect. PQST) + 2 (Area of semi-circular region with radius 35 m)

$= (PQ \times PT) + 2 \times \frac{1}{2}\pi r^2$

$= \left[(200 \times 70) + \frac{22}{7} \times 35 \times 35\right]$ m^2 = 17850 m^2.

(ii) Diameter of each outer semi-circle of the track = AE = (PT + 7 + 7) m = 84 m.

$\therefore$ Radius of each one of them = 42 m.

Outer perimeter ABCDEF = (AB + DE + semi-circle BCD + semi-circle EFA)

= (2PQ + 2 × circumference of semi-circle with radius 42 m)

$= (2 \times 200 + 2 \times \pi \times 42)$ m

$= \left[2 \times 200 + 2 \times \frac{22}{7} \times 42\right]$ m = 664 m.

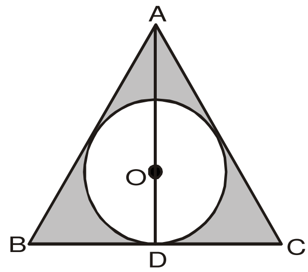
Ex.18 In an equilateral triangle of side 24 cm, a circle is inscribed, touching its sides. Find the area of the remaining portion of the triangle. Take $\sqrt{3}$ = 1·73 and π = 3·14.

Sol. Let ΔABC be the given equilateral triangle in which a circle is inscribed.

Side of the triangle, a = 24 cm.

Height of the triangle,

$h = \left(\frac{\sqrt{3}}{2} \times a\right)$ cm $= \left(\frac{\sqrt{3}}{2} \times 24\right)$ cm $= 12\sqrt{3}$ cm.



Radius of the incircle, $r = \frac{1}{3}h$

$= \left(\frac{1}{3} \times 12\sqrt{3}\right)$ cm $= 4\sqrt{3}$ cm.

$\therefore$ Required Area = Area of the shaded region

= (Area of ΔABC) – (Area of incircle)

$= \left(\frac{\sqrt{3}}{4} \times 24 \times 24 - \pi \times 4\sqrt{3} \times 4\sqrt{3}\right)$ cm^2

$= (144\sqrt{3} - 3{\cdot}14 \times 48)$ cm^2 $= (144 \times 1{\cdot}73 - 3{\cdot}14 \times 48)$ cm^2

$= [48 \times (3 \times 1.73 - 3.14)]$ cm^2

$= (48 \times 2.05)$ cm^2 = 98·4 cm^2

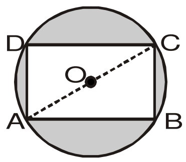
Ex.19 In the given figure, a circle circumscribes a rectangle with sides 12 cm and 9 cm. Calculate :

(i) The circumference of the circle to nearest cm,

(ii) The area of the shaded region, correct to 2 places of decimal, in cm^2.

Take $\pi = 3{\cdot}14$.

Sol. Let ABCD be the rectangle with AB = 12 cm and BC = 9 cm.



$\therefore$ AC = $\sqrt{AB^2 + BC^2} = \sqrt{(12)^2 + 9^2}$

$= \sqrt{225} = 15$ cm.

Let O be the mid-point of AC.

Then, O is the centre and OA, the radius of the circum-circle.

$\therefore$ Radius, OA $= \frac{1}{2}$ AC $= \left(\frac{1}{2} \times 15\right)$cm $= 7{\cdot}5$ cm .

$\therefore$ (i) Circumference of the circle

$= 2\pi r = (2 \times 3{\cdot}14 \times 7.5)$ cm $= 47{\cdot}1$ cm.

Hence, the circumference of the circle, correct to nearest cm is 47 cm.

(ii) Area of shaded region =

(Area of the circle)–(Area of the rectangle)

$= \left[\left(3.14 \times \frac{15}{2} \times \frac{15}{2}\right) - (12 \times 9)\right] cm^2$

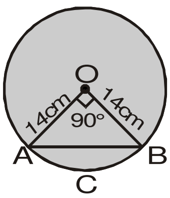
$= (176{\cdot}625 - 108)cm^2 = 68{\cdot}625 cm^2 = 68{\cdot}63 cm^2$.

Ex.20 A chord of a circle of radius 14 cm makes a right angle at the centre. Calculate :

(i) the area of the minor segment of the circle, (ii) the area of the major segment of the circle.

Sol. Let AB be the chord of a circle with centre O and radius 14 cm such that $\angle AOB = 90°$.

Thus, r = 14 cm and $\theta = 90°$.



(i) Area of sector OACB

$= \frac{\pi r^2 \theta}{360} = \left(\frac{22}{7} \times 14 \times 14 \times \frac{90}{360}\right) cm^2 = 154\ cm^2$.

Area of $\Delta OAB = \frac{1}{2} r^2 \sin\theta$

$= \left(\frac{1}{2} \times 14 \times 14 \times \sin 90°\right) cm^2 = 98\ cm^2$.

$\therefore$ Area of minor segment ACBA = (Area of sector OACB) – (Area of ΔOAB) = $(154 - 98)\ cm^2 = 56\ cm^2$.

(ii) Area of major segment BDAB

= (Area of the circle) – (Area of minor segment ACBA)

$= \left[\left(\frac{22}{7} \times 14 \times 14\right) - 56\right] cm^2 = (616 - 56) cm^2 = 560\ cm^2$.

EXERCISE – I UNSOLVED PROBLEMS

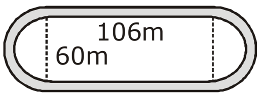
Q.1 Find the area enclosed between two concentric circles of radii 3.5 cm and 7 cm. A third concentric circle is drawn outside the 7 cm circle so that the area enclosed between it and the 7 cm circle is the same as that between the two inner circles. Find the radius of the third circle.

Q.2 Two circles touch externally. The sum of their areas is 58 π cm^2 and the distance between their centres is 10 cm. Find the radii of the two circles.

Q.3 In a circle of radius 21 cm, a chord subtends an angle of 120º at the centre of the circle. Find the area of the segment formed by the corresponding chord.

Q.4 A chord of a circle of radius 12 cm subtends an angle of 120º at the centre. Find the area of the corresponding segment of the circle. (Use π = 3.14 and $\sqrt{3}$ = 1.73)

Q.5 (a) The figure (i) given below shows a racing track whose left and right ends are semicircular. The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find (i) The distance around the track along its inner edge. (ii) the area of the track. (b) In the figure (ii) given below, the inside perimeter of a practice running track with semicircular ends and straight parallel sides is 312 m. The length of the straight portion of the track is 90 m. If the track has a uniform width of 2 m throughout, find its area.



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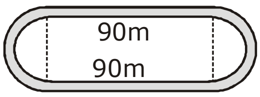
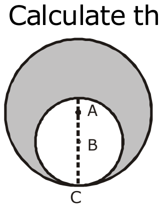


(II)

Q.6 (a) In the figure (i) given below, two circles with centres A and B touch each other at the point C. If AC = 8 cm and AB = 3 cm, find the area of the shaded region. (b) The quadrants shown in the figure (ii) given below are each of the radius 7 cm. Calculate the area of the shaded portion.



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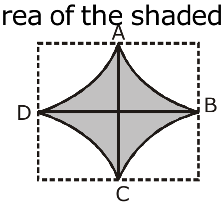
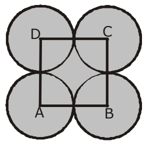


(II)

Q.7 (a) In the figure (i) given below, ABCD is a square of side 7 cm. A, B, C, and D are centres of the equal circles which touch externally in pairs. Find the area of the shaded region. (b) In the figure (ii) given below, ADC, AEB and BFC are semicircles on diameters AC, AB and BC respectively. Find the perimeter of the shaded region.



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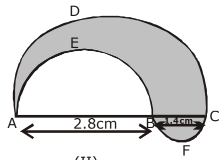
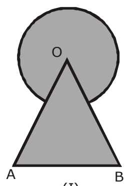


(II)

(* BC = 1.4 cm)

Q.8 (a) Find the area of the shaded region shown in figure (i) given below, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm. (b) In the figure (ii) given below, ABCD is a square. Points A, B, C and D are centres of quadrants of circles of the same radius. If the area of the shaded portion is $21\frac{3}{7}$ cm^2, find the radius of the quadrants.



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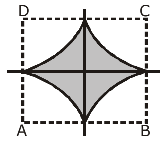
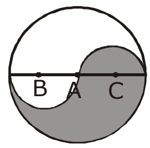


(II)

Q.9 (a) In the figure (i) given below, the points A, B and C are centres of arcs of circles of radii 5 cm, 3 cm and 2 cm respectively. Find the perimeter and the area of the shaded region. (b) In the figure (ii) given below, ABCD is square of side 4 cm. At each corner of the square a quarter circle of radius 1 cm and at the centre a circle of diameter 2 cm are drawn. Find the perimeter and the area of the shaded region. Take π = 3.14



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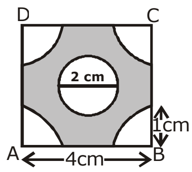
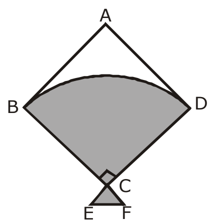


(II)

Q.10 (a) The figure (i) given below shows a kite, in which BCD is in the shape of a quadrant of a circle of radius 42 cm. ABCD is a square and ΔCEF is an isosceles right angled triangle whose equal sides are 6 cm long. Find the area of the shaded region. (b) In the figure (ii) given below, from a sheet of cardboard in the shape of a square of side 14 cm, a piece in the shape of the letter B is cut off. The curved side of the letter consists of two equal semicircles and the breadth of the rectangular piece is 1 cm. Find the area of the remaining part of the cardboard.



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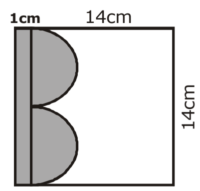


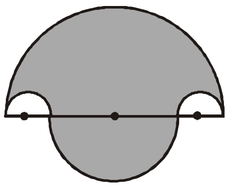
(II)

Q.11 (a) In the figure (i) given below, the boundary of the shaded region in the given diagram consists of four semi-circular arcs, the smallest two being equal. If the diameter of the largest is 14 cm and of the smallest is 3.5 cm, calculate

(i) the length of the boundary.

(ii) the area of the shaded region.

(b) In the figure (ii) given below, a piece of cardboard, in the shape of a trapezium ABCD, and AB || DC and $\angle BCD = 90^o$ quarter circle BFEC is removed. Given AB = BC = 3.5 cm and DE = 2 cm. Calculate the area of the remaining piece of the cardboard.



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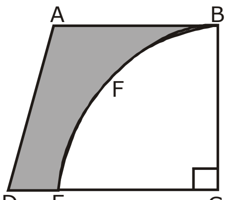
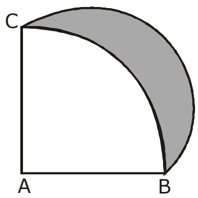


(II)

Q.12 (a) In the figure (i) given below, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region. (b) In the figure (ii) given below, ABC is a right angled triangle $\angle B = 90^o$, AB = 28 cm and BC = 21 cm. With AC as diameter a semicircle is drawn and with BC as radius a quarter circle is drawn. Find the area of the shaded region.



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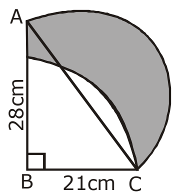
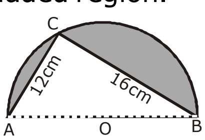


(II)

Q.13 (a) In the figure (i) given below, O is the centre of a circular arc and AOB is a straight line. Find the perimeter and the area of the shaded region correct to one decimal place. (Take π = 3.142). (b) In the figure (ii) given below, ABC is a right triangle, $\angle A = 90^o$, AB = 21 cm and AC = 28 cm. Semicircles are described on AB, BC and AC as diameters. Find the area of the shaded region.



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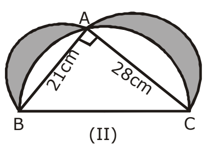
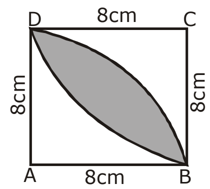


(II)

Q.14 (a) Find the area of the shaded region between the two quadrants of circles of radius 8 cm each shown in figure (i) given below, (b) In the figure (ii) given below, a circle is inscribed in an equilateral triangle. If the area of the circle is 154 cm², find the perimeter of the triangle. $\left(\text{Take } \sqrt{3} = 1.73\right)$



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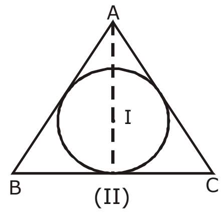
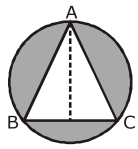


(II)

Q.15 (a) In figure (i) given below, ABC is an equilateral triangle inscribed in a circle of radius 32 cm. Find the area of the shaded region.

(b) In the figure (ii) given below, ABC is an equilateral triangle inscribed in a circle of radius 4 cm with centre O. Find the area of the shaded region.



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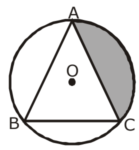


(II)

ANSWER KEY

1. $115.5\ cm^2$, $\dfrac{\sqrt{343}}{2}$ cm **2.** 7 cm, 3 cm

3. $\dfrac{21}{4}(88 - 21\sqrt{3})\,cm^2$ **4.** $88.44\ cm^2$

5. (a) (i) $400\dfrac{4}{7}$ m (ii) $4320\ m^2$ (b) $636\dfrac{4}{7}\ m^2$

6. (a) $\dfrac{858}{7}\,cm^2$ (b) $42\ cm^2$

7. (a) $164.5\ cm^2$ (b) 13.2 cm

8. (a) $\left(36\sqrt{3} + \dfrac{660}{7}\right) cm^2$ (b) 5 cm

9. (a) $\dfrac{220}{7}$ cm, $\dfrac{220}{7}\,cm^2$ (b) 20.56 cm, $9.72cm^2$

10. (a) $1404\ cm^2$ (b) $143.5\ cm^2$

11. (a) (i) 44 cm (ii) $86.625\ cm^2$ (b) $6.125\ cm^2$

12. (a) $98\ cm^2$ (b) $428.75\ cm^2$

13. (a) 59.4 cm ; $61.1\ cm^2$ (b) $35\ cm^2$

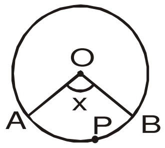
14. (a) $\dfrac{256}{7}\,cm^2$ (b) 72.66 cm

15. (a) $\left(\dfrac{22528}{7} - 768\sqrt{3}\ cm^2\right)$

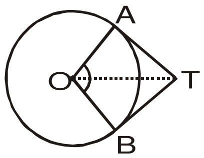
(b) $\dfrac{4}{3}\left(\dfrac{88}{7} - 3\sqrt{3}\right) cm^2$

EXERCISE – II BOARD PROBLEMS

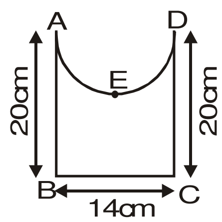
Q.1 In the fig, O is the centre of a circle. The area of sector OAPB is $\frac{5}{18}$ of the area of the circle. Find x. **[Delhi-2008]**



Q.2 In fig., if $\angle ATO = 40°$, find $\angle AOB$. **[AI-2008]**



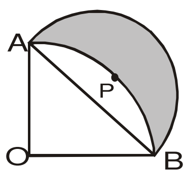
Q.3 Find the perimeter of the given figure, where $\overset{\frown}{AED}$ is a semi circle and ABCD is a rectangle. **[AI-2008]**



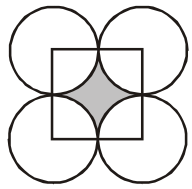
Q.4 If the diameter of a semicircular protractor is 14 cm, then find it's perimeter. **[AI-2009]**

Q.5 The length of the minute hand of a wall clock is 7 cm. How much area does it sweep in 20 minutes? **[Foreign-2009]**

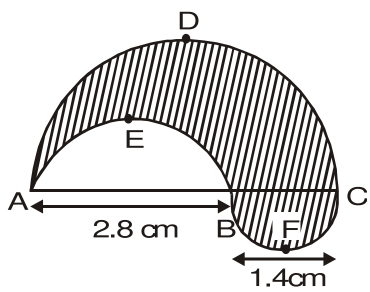
Q.6 In fig., AOBPA is a quadrant of a circle of radius 14 cm. A semicircle with AB as diameter is drawn. Find the area of the shaded region : **[Delhi-2007]**



Q.7 Four circles are described about the four corners of a square so that each touches two of the others as shown in fig. Find the area of the shaded region, each side of the square is 14 cm. (Take π = 22/7) **[Delhi-2007]**

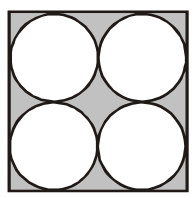


Q.8 In the fig., find the perimeter of shaded region where ADC, AEB and BFC are semicircles on diameters AC, AB and BC respectively.

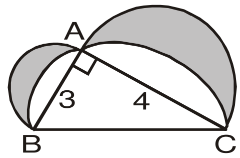


OR

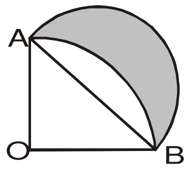
Find the area of the shaded region in the fig., where ABCD is a square of side 14 cm. **[Delhi-2008]**



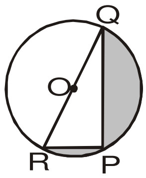
Q.9 In fig., ABC is a right-angled triangle, right-angled at A. Semicircles are drawn on AB, AC and BC as diameters. Find the area of the shaded region. **[AI-2008]**



Q.10 In the fig., ABC is a quadrant of a circle of radius 14 cm and a semi-circle is drawn with BC as diameter. Find the area of the shaded region. **[Foreign-2008]**

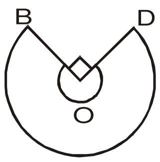


Q.11 In fig., PQ = 24 cm, PR = 7 cm and O is the centre of the circle. Find the area of shaded region. (Take π = 3.14) **[Delhi-2009]**



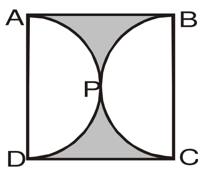
Q.12 In figure, the shape of the top of a table in a restaurant is that of a sector of a circle with centre O and $\angle BOD = 90°$. If BO = OD = 60 cm, find.

(i) the area of the top of the table
(ii) The perimeter of the table top. (Take π = 3.14)



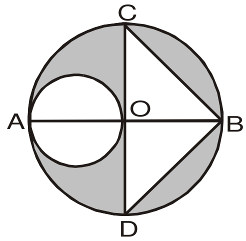
OR

In fig., ABCD is a square of side 14 cm and APD and BPC are semicircles. Find the area of shaded region.

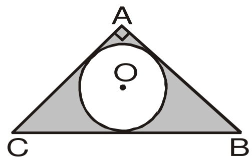


(Take π = 22/7) **[Foreign-2009]**

Q.13 In fig., AB and CD are two perpendicular diameters of a circle with centre O. If OA = 7 cm. find the area of the shaded region. (Take π = 22/7) **[AI-2010]**



Q.14 In fig., ABC is a right triangle right angled at A. Find the area of shaded region if AB = 6 cm, BC = 10 cm and O is the centre of the in circle of ΔABC. (Take π = 3.14) **[Delhi-2009]**



Q.15 The area of an equilateral triangle is $49\sqrt{3}\ cm^2$. Taking each angular point as centre, circles are drawn with radius equal to half the length of the side of the triangle. Find the area of triangle not included in the circles. (Take $\sqrt{3}$ = 1.73) **[AI-2009]**

ANSWER KEY

1.	100°	**2.**	100°
3.	$(7\pi + 54)$ cm	**4.**	36 cm
5.	$\frac{154}{3} cm^2$	**6.**	98 cm^2
7.	42 cm^2		
8.	13.2 cm OR 42 cm^2	**9.**	6 sq. units
10.	98 cm^2	**11.**	161.3 cm^2
12.	(i) 8478 cm^2 (ii) 402.6 cm OR 42 cm^2		
13.	66.5 cm^2	**14.**	11.44 cm^2
15.	7.77 cm^2		

EXERCISE – III MULTIPLE CHOICE QUESTIONS

Q.1 The diameters of two circles are 38 cm and 18 cm. Then the diameter of the circle where circumference is equal to the sum of the circumferences of two circles is :

(A) 5.6 cm (B) 28 cm
(C) 56 cm (D) 112 cm

Q.2 The radii of two circles are 4 cm and 3 cm. Then the radius of the circle whose area is equal to the sum of the area of the two circles is

(A) 7 cm (B) 5 cm
(C) 10 cm (D) 14 cm

Q.3 The sum of circumference of two circles is 132 cm. If radius of one circle is 14 cm, then radius of the second circle is

(A) 7 cm (B) 5 cm
(C) 14 cm (D) 10 cm

Q.4 A wheel has diameter 84 cm. The number of complete revolutions it will take to cover 792 metres is

(A) 100 (B) 150
(C) 200 (D) 300

Q.5 The diameter of a wheel of a bus is 140 cm. The number of revolutions the wheel will make in one minute to keep the speed of the bus 66 km/h is

(A) 100 (B) 125
(C) 250 (D) 300

Q.6 If the circumference of a circle exceeds its diameter by 16.8 cm, then the diameter of the circle is

(A) 7 cm (B) 7.84 cm
(C) 3.92 cm (D) 3.5 cm

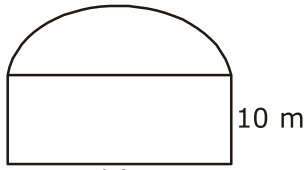
Q.7 If the perimeter of a protractor (semi-circular) is 66 cm, then its radius is

(A) 7 cm (B) 14 cm
(C) 42 cm (D) 21 cm

Q.8 A race track is in the form of a ring whose outer and inner circumference are 396 m and 352 m respectively. Then the width of the track is

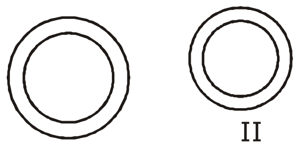
(A) 63 m (B) 56 m
(C) 7 m (D) 3.5 m

Q.9 The perimeter of the adjoining figure is



(A) 66 m (B) 56 m
(C) 70 m (D) 78

Q.10 In the adjoining figure, the inner and outer diameters of ring I are 32 cm and 34 cm respectively and those of ring II are 19 cm and 21 cm. Then the total area (in sq cm) of the two rings is



(A) 32 π (B) 20 π
(C) 52 π (D) 53π

Q.11 The area (in sq cm) of a sector whose radius is 18 cm and angle measure is 30º is

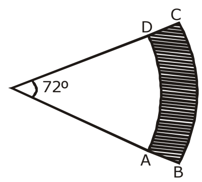
(A) 3 π (B) 18 π
(C) 27 π (D) 54 π

Q.12 The length of an arc of a circle with radius 12 cm is 10 π cm. The angle measure of this arc is

(A) 120 (B) 60
(C) 75 (D) 150

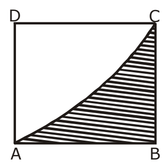
Q.13 The area (in sq cm) of the shaded region in the adjoining figure, where O is the centre of the circle, OA = 15 cm, OB = 20 cm and $\angle AOD = 72^o$, is

(A) 2 π
(B) 35 π
(C) 125 π
(D) 33 π



Q.14 ABCD is a square of side 10 cm. The area of the shaded region (in sq cm) when π = 3.142 correct to a decimal place is

(A) 21.4
(B) 85.8
(C) 21.5
(D) 78.6



Q.15 The perimeter of circular field is 242 m. The area of the field is

(A) 9317 m^2 (B) 18634 m^2
(C) 4658.5 m^2 (D) none of these

Q.16 The area of a circle is 38.5 cm². The circumference of the circle is

(A) 6.2 cm (B) 12.1 cm
(C) 11 cm (D) 22 cm

Q.17 The area of a circle is 49 π cm². Its circumference is

(A) 7 π cm (B) 14 π cm
(C) 21 π cm (D) 28 π cm

Q.18 The difference between the circumference and radius of a circle is 37 cm. The area of the circle is

(A) 111 cm² (B) 184 cm²
(C) 154 cm² (D) 259 cm²

Q.19 The circumferences of two circles are in the ratio 2 : 3. The ratio between their area is

(A) 2 : 3 (B) 4 : 9
(C) 9 : 4 (D) none of these

Q.20 On increasing the diameter of a circle by 40%, its area will be increased by

(A) 40% (B) 80%
(C) 96% (D) 82%

Q.21 On decreasing the radius of a circle by 30%, its area is decreased by

(A) 30% (B) 60%
(C) 45% (D) none of these

Q.22 The area of a square is the same as the area of a circle. Their perimeters are in the ratio

(A) 1 : 1 (B) 2 : π
(C) π : 2 (D) $\sqrt{\pi}$: 2

Q.23 The area of two circles are in the ratio 4 : 9. The ratio of their circumferences is

(A) 2: 3 (B) 3 : 2
(C) 4 : 9 (D) 9 : 4

Q.24 In making 1000 revolutions, a wheel covers 88 km. The diameter of the wheel is

(A) 14 m (B) 24 m
(C) 28 m (D) 40 m

Q.25 The diameter of a wheel is 40 cm. How many revolutions will it make in covering 176 m?

(A) 140 (B) 150
(C) 160 (D) 166

Q.26 The radius of a wheel is 0.25 m. How many revolutions will it make in covering 11 km ?

(A) 2800 (B) 4000
(C) 5500 (D) 7000

Q.27 The circumference of a circle is equal to the sum of the circumferences of two circles having diamters 36 cm and 20 cm. The radius of the new circle is

(A) 16 cm (B) 28 cm
(C) 42 cm (D) 56 cm

Q.28 The area of a circle is equal to the sum of the areas of two circles of radii 24 cm and 7 cm. The diameter of the new circle is

(A) 25 cm (B) 31 cm
(C) 50 cm (D) 62 cm

Q.29 If the sum of the areas of two circles with radii R_1 and R_2 is equal to the area of a circle of radius R, then

(A) $R_1 + R_2 = R$ (B) $R_1 + R_2 < R$
(C) $R_1^2 + R_2^2 < R^2$ (D) $R_1^2 + R_2^2 = R^2$

Q.30 If the sum of the circumferences of two circles with radii R_1 and R_2 is equal to the circumference of a circle of radius R, then

(A) $R_1 + R_2 = R$ (B) $R_1 + R_2 > R$
(C) $R_1 + R_2 < R$ (D) none of these

Q.31 If the perimeter of a square is equal to the circumference of circle, then the ratio of their areas $\left(\text{when } \pi = \frac{22}{7}\right)$ is

(A) 14 : 11 (B) 11 : 14
(C) 22 : 7 (D) 7 : 22

Q.32 If the circumference of a circle and the perimeter of a square are equal then

(A) area of the circle = area of the square
(B) (area of the circle) > (area of the square)
(C) (area of the circle) < (area of the square)
(D) none of these

Q.33 The area of the sector of a circle of radius R making a central angle of x° is

(A) $\frac{x}{180} \times 2\pi R$ (B) $\frac{x}{360} \times 2\pi R$
(C) $\frac{x}{180} \times \pi R^2$ (D) $\frac{x}{360} \times \pi R^2$

Q.34 The length of an arc of the sector of a circle of radius R making a central angle of x° is

(A) $\frac{2\pi Rx}{180}$ (B) $\frac{2\pi Rx}{360}$

(C) $\frac{\pi R^2 x}{180}$ (D) $\frac{\pi R^2 x}{360}$

Q.35 A chord of a circle of radius 28 cm subtends an angle of 45° at the centre of the circle. The area of the minor segment is $\left(\text{Take } \sqrt{2} = 1.414\right)$

(A) 30.256 cm^2 (B) 30.356 cm^2
(C) 30.456 cm^2 (D) 30.856 cm^2

Q.36 A chord of a circle subtends an angle 60° at the centre of the circle. If the length of the chord is 10 cm, then the area of the major segment is (Given π = 3.14 and $\sqrt{3}$ = 1.732)

(A) 305 cm^2 (B) 295 cm^2
(C) 310 cm^2 (D) 335 cm^2

Q.37 The perimeter of a sector of a circle with central angle 90° is 25 cm. The area of the minor segment of the circle is

(A) 14 cm^2 (B) 16 cm^2
(C) 18 cm^2 (D) 24 cm^2

Q.38 The radii of two concentric circles are 19 cm and 16 cm respectively. The area of the ring enclosed by these circles is

(A) 320 cm^2 (B) 330 cm^2
(C) 332 cm^2 (D) 340 cm^2

Q.39 The areas of two concentric circles are 1386 cm^2 and 962.5 cm^2. The width of the ring is

(A) 2.8 cm (B) 3.5 cm
(C) 4.2 cm (D) 3.8 cm

Q.40 If the perimeter and the area of a circle are numerically equal, then that radius of the circle is

(A) 2 units (B) π units
(C) 4 units (D) 7 units

Q.41 If the circumference of a circle is 0.28π cm, then its diameter is equal to

(A) 0.14 cm (B) 0.28 cm
(C) 0.56 (D) 28 cm

Q.42 If the circumference of a circle is 35.2 cm, then the radius of the circle is equal to

(A) 5.6 cm (B) 7 cm
(C) 11.2 cm (D) 14 cm

Q.43 If the radius of a circle is $\frac{7}{\sqrt{\pi}}$ cm, then the area of the circle is equal to

(A) $\frac{49}{\pi}$ cm^2 (B) 22 cm^2
(C) 49 cm^2 (D) 154 cm^2

Q.44 If the area of a circle is 2464 m^2, then its diameter is equal to

(A) 28 m (B) 56 m
(C) 154 m (D) 56 m

Q.45 If the circumference of a circle is 88 cm, then the area of the cirlce is equal to

(A) 616 cm^2 (B) 1232 cm^2
(C) 2464 cm^2 (D) 4928 cm^2

Q.46 If the area of a circle of radius r is A and its circumference is C, then

(A) A = rC (B) AC = $4r^2$
(C) AC = 2r (D) A = $\frac{1}{2}$rC

Q.47 If the ratio of the circumferences of two circles is 4 : 9, then the ratio of their areas is

(A) 9 : 4 (B) 4 : 9
(C) 2 : 3 (C) 16 : 81

Q.48 If the ratio of the areas of two circles is 25 : 16, then the ratio of their circumferences is

(A) 16 : 25 (B) 4 : 5
(C) 5 : 4 (D) 625 : 256

Q.49 If the sum of the circumferences of two circles with diameters d_1 and d_2 is equal to the circumference of a circle of diameter d, then

(A) $d_1^2 + d_2^2 = d^2$ (B) $d_1 + d_2 = d$
(C) $d_1 + d_2 > d$ (D) $d_1 + d_2 < d$

Q.50 If the sum of the area of two circles with radii r_1 and r_2 is equal to the area of a circle of radius r, then

(A) $r_1^2 + r_2^2 = r^2$ (B) $r_1 + r_2 = r$
(C) $r_1^2 + r_2^2 > r^2$ (D) $r_1^2 + r_2^2 < r^2$

Q.51 If the area of a circle is equal to the sum of areas of two circles with diameters 10 cm and 24 cm, then the diameter of the circle is

(A) 34 cm (B) 26 cm

(C) 17 cm (D) 13 cm

Q.52 If the radius of a bicycle wheel is 35 cm, then the distance covered by the bicycle in 50 complete revolutions is equal to

(A) 440 m (B) 220 m

(C) 110 m (D) 350 m

Q.53 If a bicycle wheel makes 5000 revolutions in moving 11 km, then the diameter of the wheel is

(A) 35 cm (B) 70 cm

(C) 1.4 m (D) 70 m

Q.54 If the diameters of two concentric circles are 20 cm and 16 cm, then the area of the circular ring included between the two circles is

(A) 400 π cm^2 (B) 256 π cm^2

(C) 144 π cm^2 (D) 36 π cm^2

Q.55 If a circular grass lawn of 35 m in radius has a path 7 m wide running around it on the outside, then the area of the path is

(A) 1450 m^2 (B) 1576 m^2

(C) 1694 m^2 (D) 3368 m^2

Q.56 If the difference between circumference and radius of a circle is 37 cm, then the circumference of that circle is

(A) 154 cm (B) 44 cm

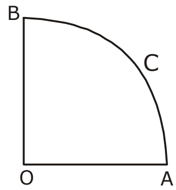
(C) 14 cm (D) 7 cm

Q.57 A wire is in the form of a circle of radius 14 cm, then the side of the square into which it can be bent is

(A) 22 cm (B) 2 πcm

(C) (π + 14) cm (D) 11 cm

Q.58 In the adjoining figure, OACB is a quadrant of a circle of radius 7 cm. Then perimeter of the quadrant is



(A) 11 cm (B) 18 cm

(C) 25 cm (D) 36 cm

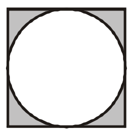
Q.59 In the adjoining figure, a circle is inscribed in a square of side 14 cm. The area of the shaded region is equal to

(A) 196 cm^2

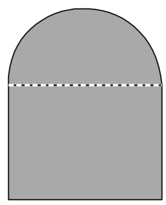
(B) 154 cm^2

(C) 52 cm^2

(D) 42 cm^2



Q.60 The adjoining figure shown a rectangle and a semicircle. The perimeter of the shaded region is



(A) 70 cm (B) 56 cm

(C) 78 cm (D) 46 cm

Q.61 The area of the shaded region shown in the above question is

(A) 140 cm^2 (B) 77 cm^2

(C) 294 cm^2 (D) 217 cm^2

Q.62 If the sector of a circle of diameter 14 cm subtendes and angle of 30º at the centre, then its area is

(A) 154 cm^2 (B) 77 cm^2

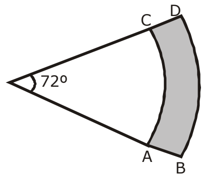
(C) $\frac{77}{3}$ cm^2 (D) $\frac{77}{6}$ cm^2

Q.63 If the sector of a circle of diameter 10 cm subtends an angle of 144º at the centre, then the length of the arc of the sector is

(A) 2π cm (B) 4 π cm

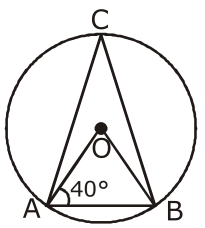
(C) 5 π cm (D) 6 π cm

Q.64 In the adjoining figure, O is the centre of a circle. If OA = 10 cm, OB = 15 cm and $\angle BOD = 72^\circ$, then the area of the shaded region is



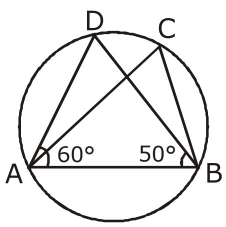
(A) 5 πcm^2
(B) 10 πcm^2
(C) 25 πcm^2
(D) 35 πcm^2

Q.65 In figure, if $\angle OAB = 40^\circ$, then $\angle ACB$ is equal to: **[NTSE]**



(A) 50° (B) 40°
(C) 60° (D) 70°

Q.66 In figure, if $\angle DAB = 60^\circ$, $\angle ABD = 50^\circ$, then $\angle ACB$ is equal to: **[NTSE]**

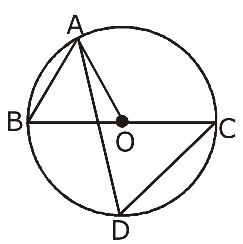


(A) 60° (B) 50°
(C) 70° (D) 80°

Q.67 ABCD is a cyclic quadrilateral such that AB is a diameter of the circle circumscribing it and $\angle ADC = 140^\circ$, then $\angle BAC$ is equal to: **[NTSE]**

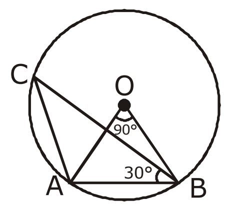
(A) 80° (B) 50°
(C) 40° (D) 30°

Q.68 In figure, BC is a diameter of the circle and $\angle BAO = 60^\circ$. Then $\angle ADC$ is equal to: **[NTSE]**



(A) 30° (B) 45°
(C) 60° (D) 120°

Q.69 In figure, $\angle AOB = 90^\circ$ and $\angle ABC = 30^\circ$, then $\angle COA$ is equal to: **[NTSE]**



(A) 30° (B) 45°
(C) 90° (D) 60°

ANSWER KEY

1.	C	**2.**	B	**3.**	A	**4.**	D
5.	C	**6.**	B	**7.**	D	**8.**	C
9.	B	**10.**	D	**11.**	C	**12.**	D
13.	B	**14.**	C	**15.**	C	**16.**	D
17.	B	**18.**	C	**19.**	B	**20.**	C
21.	D	**22.**	D	**23.**	A	**24.**	C
25.	A	**26.**	D	**27.**	B	**28.**	A
29.	D	**30.**	A	**31.**	B	**32.**	B
33.	D	**34.**	B	**35.**	D	**36.**	A
37.	A	**38.**	B	**39.**	B	**40.**	D
41.	A	**42.**	B	**43.**	C	**44.**	B
45.	C	**46.**	B	**47.**	C	**48.**	A
49.	D	**50.**	D	**51.**	B	**52.**	A
53.	C	**54.**	D	**55.**	B	**56.**	A
57.	B	**58.**	C	**59.**	B	**60.**	C
61.	C	**62.**	D	**63.**	A	**64.**	B
65.	A	**66.**	C	**67.**	B	**68.**	C
69.	D						

EXERCISE – IV NTSE /OLYMPIAD /FOUNDATION PROBLEMS

CHOOSE THE CORRECT ONE

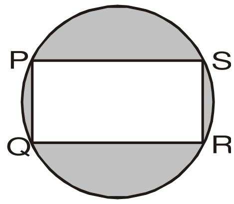
1. In the adjoining figure PQRS is a rectangle 8 cm × 6 cm, inscribed in the circle. The area of the shaded portion will be :

(A) 48 cm^2

(B) 42.50 cm^2

(C) 32.50 cm^2

(D) 30.5 cm^2



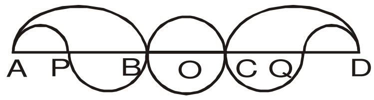
2. In the adjoining figure AB = CD = 2BC = 2BP = 2 CQ. In the middle, a circle with radius 1 cm is drawn. In the rest figure all are the semicircular arcs. What is the perimeter of the whole figure?

(A) 4 π

(B) 8π

(C) 10 π

(D) None of these



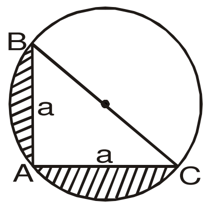
3. If BC passes through centre of the circle, then the area of the shaded region in the given figure is :

(A) $\frac{a^2}{2}(3-\pi)$

(B) $a^2\left(\frac{\pi}{2}-1\right)$

(C) $2a^2(\pi - 1)$

(D) $\frac{a^2}{2}\left(\frac{\pi}{2}-1\right)$



4. Two circles of unit radii, are so drawn that the centre of each lies on the circumference of the other. The area of the region common to both the circles, is :

(A) $\frac{(4\pi-3\sqrt{3})}{12}$ (B) $\frac{(4\pi-6\sqrt{3})}{12}$

(C) $\frac{(4\pi-3\sqrt{3})}{6}$ (D) $\frac{(4\pi-6\sqrt{3})}{6}$

5. The area of the largest possible square inscribed in a circle of unit radius (in square unit) is :

(A) 3 (B) 4 (C) $2\sqrt{3}\pi$ (D) 2

6. The area of the largest triangle that can be inscribed in a semicircle of radius r is :

(A) r^2 cm^2 (B) $\left(\frac{r}{3}\right)^2 cm^2$ (C) $r\sqrt{2}\, cm^2$ (D) $3\sqrt{3}r\, cm^2$

7. If a regular hexagon is inscribed in a circle of radius r, then its perimeter is :

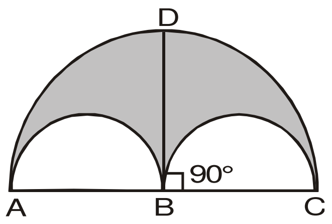
(A) $6\sqrt{3}r$ (B) 6r (C) 3r (D) 12r

8. If a regular hexagon circumscribes a circle of radius r, then its perimeter is :

(A) $4\sqrt{3}r$ (B) $6\sqrt{3}r$ (C) 6r (D) $12\sqrt{3}r$

9. In the adjoining figure there are three semicircles in which BC = 6 cm and BD = $6\sqrt{3}$ cm. What is the area

of the shaded region (in cm) :



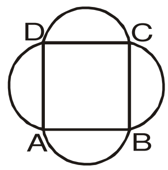
(A) 12π

(B) 9π

(C) 27π

(D) 28π

10. ABCD is a square of side a cm. AB, BC, CD and AD all are the chords of circles with equal radii each. If the chords subtends an angle of 120° at their respective centres, find the total area of the given figure, where arcs are part of the circles :



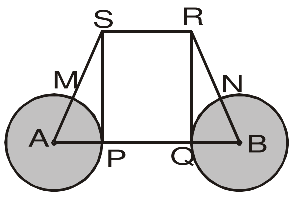
(A) $\left[a^2 + 4\left(\frac{\pi a^2}{9} - \frac{a^2}{3\sqrt{2}}\right)\right]$

(B) $\left[a^2 + 4\left(\frac{\pi a^2}{9} - \frac{a^2}{4\sqrt{3}}\right)\right]$

(C) $[9a^2 - 4\pi + 3\sqrt{3}a^2]$

(D) None of these

11. In the adjoining figure PQRS is a square and MS = RN and A, P, Q and B lie on the same line. Find the ratio of the area of two circles to the area of the square. Given that AP = MS.



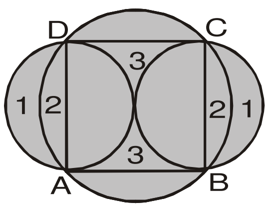
(A) $\frac{\pi}{3}$

(B) $\frac{2\pi}{3}$

(C) $\frac{3\pi}{2}$

(D) $\frac{6}{\pi}$

Direction for question number (12 to14) : In the adjoining figure ABCD is a square. A circle ABCD is passing through all the four vertices of the square. There are two more circles on the sides AD and BC touching each other inside the square, AD and BC are the respective diameters of the two smaller circles. Area of the square is 16 cm^2.



12. What is the area of region 1 ?

(A) 2.4 cm^2 (B) $\left(2 - \frac{\pi}{4}\right)$cm^2

(C) 8 cm^2 (D) $(4\pi - 2)$ cm^2

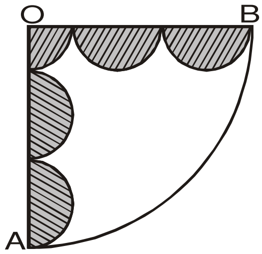
13. What is the area of region 2?

(A) $3(\pi - 2)$ cm^2 (B) $(\pi - 3)$ cm^2 (C) $(2\pi - 3)$ cm^2 (D) $4(\pi - 2)$ cm^2

14. What is the area of region 3?

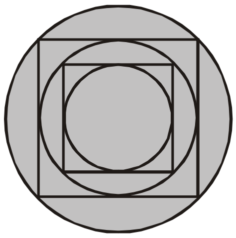
(A) $(4 - 4\pi)$ cm^2 (B) $4(4 - \pi)$ cm^2 (C) $(4\pi - 2)$ cm^2 (D) $(3\pi + 2)$ cm^2

15. A circular paper is folded along its diameter, then again it is folded to form a quadrant. Then it is cut as shown in the figure, after it the paper was reopened in the original circular shape. Find the ratio of the original paper to that of the remaining paper? (The shaded portion is cut off from the quadrant. The radius of quadrant OAB is 5 cm and radius of each semicircle is 1 cm) :



(A) 25 : 16 (B) 25 : 9 (C) 20 : 9 (D) None of these

Directions for question number 16-18 : A square is inscribed in a circle then another circle is inscribed in the square. Another square is then inscribed in the circle. Finally a circle is inscribed in the innermost square. Thus there are 3 circles and 2 squares as shown in the fig. The radius of the outer-most circle is R.



16. What is the radius of the inner-most circle?

(A) $\frac{R}{2}$ (B) $\frac{R}{\sqrt{2}}$ (C) $\sqrt{2}R$ (D) None of these

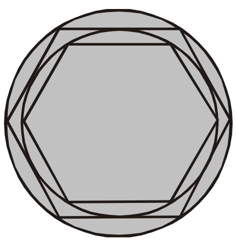
17. What is the sum of areas of all the squares shown in the figure?

(A) $3R^2$ (B) $3\sqrt{2}R^2$ (C) $\frac{3}{\sqrt{2}}R^2$ (D) None of these

18. What is the ratio of sum of circumferences of all the circles to the sum of perimeters of all the squares?

(A) $(2+\sqrt{3})\pi R$ (B) $(3+\sqrt{2})\pi R$ (C) $3\sqrt{3}\pi R$ (D) None of these

Directions for question number 19-21 : A regular hexagon is inscribed in a circle of radius R. Another circle is inscribed in the hexagon. Now another hexagon is inscribed in the second (smaller) circle.



19. What is the sum of perimeters of both the hexagons?

(A) $(2+\sqrt{3})R$ (B) $3(2+\sqrt{3})R$ (C) $3(3+\sqrt{2})R$ (D) None of these

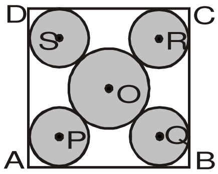
20. What is the ratio of area of inner circle to the outer circle?

(A) 3 : 4 (B) 9 : 16 (C) 3 : 8 (D) None of these

21. If there are some more circles and hexagons inscribed in the similar way as given above, then the ratio of each side of outermost hexagon (largest one) to that of the fourth (smaller one) hexagon is (fourth hexagon means the hexagon which is inside the third hexagon from the outerside.) :

(A) $9 : 3\sqrt{2}$ (B) 16 : 9 (C) $8 : 3\sqrt{3}$ (D) None of these

22. In the adjoining diagram ABCD is a square with side 'a' cm. In the diagram the area of the larger circle with centre 'O' is equal to the sum of the areas of all the rest four circles with equal radii, whose centres are P, Q, R and S. What is the ratio between the side of square and radius of a smaller circle?



(A) $(2\sqrt{2}+3)$ (B) $(2+3\sqrt{2})$ (C) $(4+3\sqrt{2})$ (D) Can't be determined.

23. There are two concentric circles whose areas are in the ratio of 9 : 16 and the difference between their diameters is 4 cm. What is the area of the outer circle?

(A) 32 cm^2 (B) 64π cm^2 (C) 36 cm^2 (D) 48 cm^2

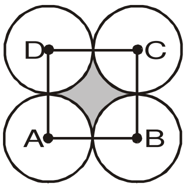
24. ABCD is a square, 4 equal circles are just touching each other whose centres are the vertices A, B, C, D of the square. What is the ratio of the shaded to the unshaded area within square?

(A) $\frac{8}{11}$

(B) $\frac{3}{11}$

(C) $\frac{5}{11}$

(D) $\frac{6}{11}$



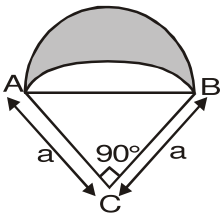
25. In the adjoining figure ACB is a quadrant with radius 'a'. A semicircle is drawn outside the quadrant taking AB as a diameter. Find the area of shaded region :

(A) $\frac{1}{4}(\pi - 2a^2)$

(B) $\left(\frac{1}{4}\right)(\pi a^2 - a^2)$

(C) $\frac{a^2}{2}$

(D) Can't be determined



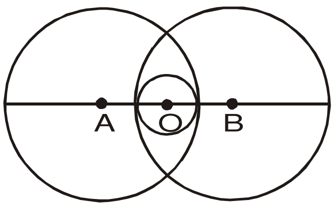
26. There are two circles intersecting each other. Another smaller circle with centre O, is lying between the common region of two larger circles. Centre of the circle (i.e., A, O and B) are lying on a straight line. AB = 16 cm and the radii of the larger circles are 10 cm each. What is the area of the smaller circle?

(A) 4π cm^2

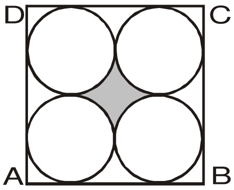
(B) 2π cm^2

(C) $\frac{4}{\pi}$ cm^2

(D) $\frac{\pi}{4}$ cm^2



27. ABCD is a square, inside which 4 circles with radius 1 cm, each are touching each other. What is the area of the shaded region?



(A) $(2\pi - 3)$ cm^2

(B) $(4 - \pi)$ cm^2

(C) $(16 - 4\pi)$ cm^2

(D) None of these

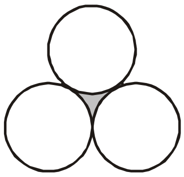
28. Three circles of equal radii touch each other as shown in figure. The radius of each circle is 1 cm. What is the area of shaded region?

(A) $\left(\frac{2\sqrt{3}-\pi}{2}\right)$cm^2

(B) $\left(\frac{3\sqrt{2}-\pi}{3}\right)$cm^2

(C) $\frac{2\sqrt{3}}{\pi}$cm^2

(D) None of these



OBJECTIVE						ANSWER KEY								EXERCISE-4
Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	D	C	D	C	D	A	B	A	C	B	B	C	D	B	A
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28		
Ans.	A	A	D	B	A	C	B	B	B	C	A	B	A		

SURFACE AREAS AND VOLUMES

INTRODUCTION

In this chapter, we shall discuss problems on conversion of one of the solids like cuboid, cube, right circular cylinder, right circular cone and sphere in another.

In our day-to-day life we come across various solids which are combinations of two or more such solids. For example, a conical circus tent with cylindrical base is a combination of a right circular cylinder and a right circular cone, also an ice-cream cone is a combination of a cone and a hemi-sphere. We shall discuss problems on finding surface areas and volumes of such solids. We also come across solids which are a part of a cone. For example, a bucket, a glass tumbler, a friction clutch etc. These solids are known as frustums of a cone. In the end of the chapter, we shall discuss problems on surface area and volume of frustum of a cone.

UNITS OF MEASUREMENT OF AREA AND VOLUME

The inter-relationships between various units of measurement of length, area and volume are listed below for ready reference:

LENGTH

1 Centimetre (cm) = 10 milimetre (mm)

1 Decimetre (dm) = 10 centimetre

1 Metre(m) = 10 dm = 100 cm = 1000mm

1 Decametre (dam) = 10 m = 1000 cm

1 Hectometre (hm) = 10 dam = 100 m

1 Kilometre (km) = 1000 m = 100 dam = 10 hm

1 Myriametre = 10 kilometre

AREA

1 cm^2 = 1 cm × 1 cm = 10 mm × 10 mm = 100 mm^2

1 dm^2 =1 dm × 1 dm =10 cm × 10 cm =100 cm^2

1 m^2 = 1 m × 1 m=10 dm × 10 dm =100 dm^2

1 dam^2 =1 dam × 1 dam = 10 m × 10 m = 100 m^2

1 hm^2 = 1 hectare = 1 hm × 1 hm = 100 m × 100 m = 10000 m^2 = 100 dm^2

1 km^2 = 1 km × 1 km = 10 hm × 10 hm = 100 hm^2 or 100 hectare

VOLUME

1 cm^3 = 1 ml = 1 cm × 1 cm × 1 cm = 10 mm × 10 mm × 10 mm = 1000 mm^3

1 litre = 1000 ml = 1000 cm^3

1 m^3 = 1 m × 1 m × 1 m = 100 cm × 100 cm × 100 cm = 10^6 cm^3 = 1000 litre = 1 kilolitre

1 dm^3 = 1000 cm^3

1 m^3 = 1000 dm^3

1 km^3 = 10^9 m^3

BASIC CONCEPTS AND IMPORTANT RESULTS

* **Solid**

*A figure which occupies a portion of space by plane of curved surfce is called a **solid.*** A solid has three dimension – length, breadth and height.

* **Volume of a solid**

The measurement of the space enclosed by a solid is called its **volume.** It is measured in cubic units.

* **Surface area of a solid**

The sum of the areas of the plane or curved surfaces (faces) of a solid is called its **(total) *surface area.***

It is measured in square units.

Total surface area (abbreviated TSA) of a solid

= curved surface area (abbreviated CSA) + area of plane faces (if any).

* **Surface area and volume of solids**

1. Cuboid

Let l, b and h be the length, breadth and height of a cuboid respectively, then

(i) lateral surface area = $2h(l + b)$

(ii) total surface area = $2(lb + bh + hl)$

(iii) volume = lbh.

2. Cube

Let a be the length of an edge of a cube, then

(i) lateral suface area = $4a^2$

(ii) total surface area = $6a^2$

(iii) volume = a^3.

3. Solid cylinder

Let r be the radius and h be the height of a solid cylinder, then

(i) curved (lateral) surface area = $2\pi rh$

(ii) total surface area = $2\pi r(h + r)$

(iii) volume = $\pi r^2 h$.

4. Hollow cylinder

Let R and r be the external and internal radii, and h be the height of a hollow cylinder, then

(i) the thickness of cylinder = $R - r$

(ii) external curved surface area = $2\pi Rh$

(iii) internal curved surface area = $2\pi rh$

(iv) total surface area= $2\pi(Rh + rh + R^2 - r^2)$

(v) volume of material = $\pi(R^2 - r^2)h$.

5. Cone

Let r, h and l be the radius, height and slant height respectively of a circular cone, then

(i) slant height = $l = \sqrt{r^2 + h^2}$

(ii) curved (lateral) surface area = πrl

(iii) total surface area = $\pi r(l + r)$

(iv) volume = $\frac{1}{3}\pi r^2 h$

6. Solid sphere

Let r be the radius of a solid sphere, then

(i) surface area = $4\pi r^2$

(ii) volume = $\frac{4}{3}\pi r^3$.

7. Spherical shell

Let R and r be the radii of the outer and inner spheres, then

(i) thickness of the shell = R - r

(ii) volume of material = $\frac{4}{3}\pi(R^3 - r^3)$.

8. Solid hemisphere

Let r be the radius of a hemisphere, then

(i) curved (lateral) surface area = $2\pi r^2$

(ii) total surface area = $3\pi r^2$

(iii) volume = $\frac{2}{3}\pi r^3$.

9. Hemisperical shell

Let R and r be the radii of the outer and inner hemispheres, then

(i) the thickness of the shell = R - r

(ii) external curved surface area = $2\pi R^2$

(iii) internal curved surface area = $2\pi r^2$

(iv) total surface area = $\pi(3R^2 + r^2)$

(v) volume of material = $\frac{2}{3}\pi(R^3 - r^3)$.

10. Frustum of a cone

Let R, r be the radii of the circular bases of the frustum, h and l be its height and slant height respectively, then

(i) slant height = l = $\sqrt{h^2 + (R-r)^2}$

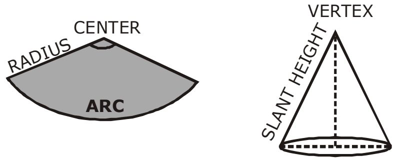
(ii) $\pi(R + r)\,l$ = curved (lateral) surface area

(iii) total (whole) surface area = $\pi[(R + r)\,l + R^2 + r^2]$

(iv) volume = $\frac{1}{3}\pi h(R^2 + r^2 + Rr)$

*** Remark**

If a sector of a circle is folded to make the two bounding radii coincide, the surface so formed is a *hollow right circular cone.*



We note the following :

(i) The vertex of the cone is the centre of the circle.

(ii) The slant height of the cone is equal to the radius of the circle.

SOLVED PROBLEMS

Ex.1 The length, breadth and height of a rectangular solid are in the ratio 6 : 5 : 4. If the total surface area is 5328 cm^2, find the length, breadth and height of the solid.

Sol. Let length = (6x) cm, breadth = (5x) cm and height = (4x) cm.

Then, total surface area

$= [2(6x \times 5x + 5x \times 4x + 4x \times 6x)]\ cm^2$

$= [2(30x^2 + 20x^2 + 24x^2)]\ cm^2 = (148x^2)\ cm^2$.

$\therefore\ 148x^2 = 5328 \Rightarrow x^2 = \frac{5328}{148} = 36 \Rightarrow x = 6$.

Hence, length = 36 cm, breadth = 30 cm, height = 24 cm.

Ex.2 An open rectangular cistern is made of iron 2·5 cm thick. When measured from outside, it is 1 m 25 cm long, 1 m 5 cm broad and 90 cm deep.

Find: (i) the capacity of the cistern in litres;

(ii) the volume of iron used;

(iii) the total surface area of the cistern.

Sol. External dimensions of the cistern are :

Length = 125 cm, Breadth = 105 cm and Depth = 90 cm.

Internal dimensions of the cistern are :

Length = 120 cm, Breadth = 100 cm and Depth = 87·5 cm.

(i) Capacity = Internal volume

$= (120 \times 100 \times 87.5)\ cm^3 = \left(\frac{120 \times 100 \times 87.5}{1000}\right)$ litres = 1050 litres.

(ii) Volume of iron = (External volume) – (Internal volume) = $[(125 \times 105 \times 90) - (120 \times 100 \times 87{\cdot}5)]\ cm^3$

$= (1181250 - 1050000)\ cm^3 = 131250\ cm^3$.

(iii) External area = (Area of 4 faces) + (Area of the base) = $([2(125 + 105) \times 90] + (125 \times 105)\}\ cm^2$

$= (41400 + 13125)\ cm^2 = 54525\ cm^2$.

Internal area = $([2(120 + 100) \times 87{\cdot}5] + (120 \times 100)\}\ cm^2 = (38500 + 12000)\ cm^2 = 50500\ cm^2$.

Area at the top = Area between outer and inner rectangles = $[(125 \times 105) - (120 \times 100)]\ cm^2$

$= (13125 - 12000)\ cm^2 = 1125\ cm^2$.

$\therefore$ Total surface area = $(54525 + 50500 + 1125)\ cm^2 = 106150\ cm^2$.

Ex.3. A field is 80 m long and 50 m broad. In one corner of the field, a pit which is 10 m long, 7.5 m broad and 8 m deep has been dug out. The earth taken out of it is evenly spread over the remaining part of the field. Find the rise in the level of the field.

Sol. Area of the field = $(80 \times 50)\ m^2 = 4000\ m^2$

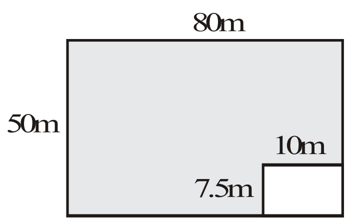
Area of the pit = $(10 \times 7.5)\ m^2 = 75\ m^2$

Area over which the earth is spread out

$= (4000 - 75)\ m^2 = 3925\ m^2$

Volume of earth dug out = $(10 \times 7.5 \times 8)\ m^3$

$= 600\ m^3$.



$\therefore$ Rise in level = $\left(\frac{\text{Volume}}{\text{Area}}\right)$

$= \left(\frac{600}{3925}\right) m = \left(\frac{600 \times 100}{3925}\right) cm = 15.3$ cm

Ex.4 A room is half as long again as it is broad. The cost of carpeting the room at Rs 18 per m^2 is Rs 972 and the cost of white-washing the four walls at Rs 6 per m^2 is Rs 1080. Find the dimensions of the room.

Sol. Let breadth = (x) m. Then, length = $\left(\frac{3}{2}x\right)$m.

Let height of the room = y m.

Area of the floor = $\left(\frac{\text{cost of carpeting}}{\text{Rate}}\right) = \left(\frac{972}{18}\right)$

$= 54\ m^2$

$\therefore x \times \frac{3}{2}x = 54 \Rightarrow x^2 = \left(54 \times \frac{2}{3}\right) = 36 \Rightarrow x = 6.$

So, breadth = 6 m and length = $\left(\frac{3}{2} \times 6\right)$ m = 9 m.

Now, area of four walls

$= \left(\frac{\text{cost of white-washing}}{\text{Rate}}\right) = \left(\frac{1080}{6}\right)m^2 = 180\ m^2.$

$\therefore 2(9+6) \times y = 180 \Rightarrow 30y = 180 \Rightarrow y = \left(\frac{180}{30}\right) = 6.$

Hence, length = 9m, breadth = 6m, height = 6m.

Ex.5 The water in a rectangular reservoir having a base 80 m × 60 m, is 6·5 m deep. In what time can the water be emptied by a pipe of which the cross section is a square of side 20 cm, if water runs through the pipe at the rate of 15 km/ hr ?

Sol. Volume of water in the reservoir = $(80 \times 60 \times 6.5)\ m^3 = 31200\ m^3$.

Area of cross section of the pipe = $\left(\frac{20}{100} \times \frac{20}{100}\right) m^2 = \frac{1}{25} m^2$.

Volume of water emptied in 1 hr = $\left(\frac{1}{25} \times 15000\right) m^3 = 600\ m^3$.

Time taken to empty the reservoir = $\left(\frac{31200}{600}\right)$ hrs = 52 hrs.

Ex.6 2.2 cu dm of brass is to be drawn into a cylindrical wire of diameter 0.50 cm. Find the length of the wire.

Sol. Volume of brass = 2.2 cu dm = $(2.2 \times 10 \times 10 \times 10)\ cm^3 = 2200\ cm^3$. Let the required length of wire be x cm.

Then, its volume = $(\pi r^2 x)\ cm^3$

$= \left(\frac{22}{7} \times 0.25 \times 0.25 \times x\right) cm^3$

$\frac{22}{7} \times 0.25 \times 0.25 \times x = 2200$

$\Rightarrow x = \left(2200 \times \frac{7}{22} \times \frac{1}{0.25 \times 0.25}\right)$

x = 11200 cm = 112 m.

Hence, the length of wire is 112 m.

Ex.7 A well with 14 m diameter is dug 8 m deep. The earth taken out of it has been evenly spread all around it to a width of 21 m to form an embankment. Find the height of the embankment.

Sol. Volume of earth dug out from the well = $\pi r^2 h = \left(\frac{22}{7} \times 7 \times 7 \times 8\right) m^3 = 1232\ m^3$.

Area of the embankment = $\pi(R^2 - r^2) = \frac{22}{7} \times \{(28)^2 - (7)^2\}\ m^2 = \left(\frac{22}{7} \times 35 \times 21\right) m^2 = 2310\ m^2$.

Height of the embankment

$= \frac{\text{Volume of earth dug out}}{\text{Area of embankment}} = \left(\frac{1232}{2310} \times 100\right) \text{cm} = 53.3 \text{ cm}.$

Ex.8 The difference between the outside and inside surface of a cylindrical metallic pipe 14 cm long is 44 cm^2. If the pipe is made of 99 cu cm of metal, find the outer and inner radii of the pipe.

Sol. Let, external radius = R cm and internal radius = r cm.

Then, outside surface = $2\pi Rh$

$= \left(2 \times \frac{22}{7} \times R \times 14\right) cm^2 = (88R)\ cm^2.$

Inside surface = $2\pi rh = \left(2 \times \frac{22}{7} \times r \times 14\right) cm^2 = (88r)\ cm^2.$

$(88R - 88r) = 44 \Rightarrow (R - r) = \frac{44}{88} = \frac{1}{2}$

$\Rightarrow (R - r) = \frac{1}{2}$ .... (i)

External volume = $\pi R^2 h = \left(\frac{22}{7} \times R^2 \times 14\right) cm^3 = (44R^2)\ cm^3$

Internal volume = $\pi r^2 h = \left(\frac{22}{7} \times r^2 \times 14\right) cm^3 = (44r^2)\ cm^3$

$(44R^2 - 44r^2) = 99 \Rightarrow (R^2 - r^2) = \frac{99}{44}$

$\Rightarrow (R^2 - r^2) = \frac{9}{4}$ .... (ii)

On dividing (ii) by (i), we get: $(R + r) = \left(\frac{9}{4} \times \frac{2}{1}\right) \Rightarrow (R + r) = \frac{9}{2}$ .... (iii)

Solving (i) and (iii), we get, R = 2.5 and r = 2.

Hence, outer radius = 2.5 cm and inner radius = 2 cm.

Ex.9 A solid iron rectangular block of dimensions 4.4 m, 2.6 m and 1 m is cast into a hollow cylindrical pipe of internal radius 30 cm and thickness 5 cm. Find the length of the pipe.

Sol. Volume of iron = (440 x 260 x 100) cm^3.

Internal radius of the pipe = 30 cm.

External radius of the pipe = (30+5)cm=35 cm.

Let the length of the pipe be h cm.

Volume of iron in the pipe = (External volume) - (Internal volume)

$= [\pi \times (35)^2 \times h - \pi \times (30)^2 \times h]\ cm^3 = (\pi h)\ \{(35)^2 - (30)^2\}\ cm^3$

$= (65 \times 5)\ \pi h\ cm^3 = (325\ \pi h)\ cm^3.$

$\therefore\ 325\pi h = 440 \times 260 \times 100$

$\Rightarrow\ h = \left(\frac{440 \times 260 \times 100}{325} \times \frac{7}{22}\right) cm$

$\Rightarrow\ h = \left(\frac{11200}{100}\right) m = 112\ m.$

Hence, the length of the pipe is 112 m.

Ex.10 A cylindrical pipe has inner diameter of 7 cm and water flows through it at 192.5 litres per minute. Find the rate of flow in kilometres per hour.

Sol. Volume of water that flows per hour = (192.50×60) litres = (192.5×60×1000) cm^3.

Inner radius of the pipe = 3.5 cm.

Let the length of column of water that flows in 1 hour be h cm.

Then, $\frac{22}{7} \times 3.5 \times 3.5 \times h = 192.5 \times 60 \times 1000$

$\Rightarrow h = \left(\frac{192.5 \times 60 \times 1000 \times 7}{3.5 \times 3.5 \times 22}\right) cm = 300000\ cm = 3\ km$

Hence, the rate of flow = 3 km per hour.

Ex.11 The total surface area of a right circular cone of slant height 13 cm is 90π cm^2.
Calculate : (i) its radius in cm, (ii) its volume in cm^3, in terms of π.

Sol. Given : slant height, ℓ = 13 cm.
Let, radius = r cm and height = h cm.

(i) Total surface area = $\pi r(\ell + r) = [\pi r(13+r)]$ cm^2.
$\therefore \pi r(13 + r) = 90\pi \Rightarrow r^2 + 13r - 90 = 0$
$\Rightarrow (r + 18)(r - 5) = 0$
$\Rightarrow r = 5$ [Neglecting r = – 18, as radius cannot be negative]
$\therefore$ Radius of the cone = 5 cm.

(ii) $h = \sqrt{\ell^2 - r^2} = \sqrt{(13)^2 - (5)^2}$
$= \sqrt{169 - 25} = \sqrt{144}$ = 12 cm.
$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h$
$= \left(\frac{1}{3} \times \pi \times 5 \times 5 \times 12\right)$ cm^3
$= 100\pi$ cm^3.

Ex.12 A girl fills a cylindrical bucket 32 cm in height and 18 cm in radius with sand. She empties the bucket on the ground and makes a conical heap of the sand. If the height of the conical heap is 24 cm, find :
(i) its radius, (ii) its slant height. **[NCERT]**

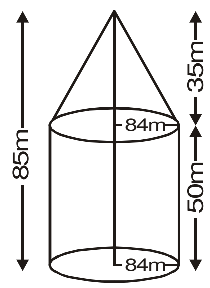
Sol. Height of cylindrical bucket, H = 32 cm.
Radius of cylindrical bucket, R = 18 cm.

Volume of sand $= \pi R^2 H = \left(\frac{22}{7} \times 18 \times 18 \times 32\right)$ cm^3.

(i) Height of conical heap, h = 24 cm.
Let the radius of the conical heap be r cm.
Then, volume of conical heap
$= \frac{1}{3}\pi r^2 h = \left(\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24\right)$ cm^3.
Now, Volume of conical heap = Volume of sand
$\Rightarrow \left(\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24\right) = \left(\frac{22}{7} \times 18 \times 18 \times 32\right)$
$\Rightarrow r^2 = \left(\frac{18 \times 18 \times 32 \times 3}{24}\right) = (18 \times 18 \times 4)$
$\Rightarrow r = \sqrt{(18 \times 18 \times 4)} = (18 \times 2)$ cm = 36 cm.
$\therefore$ Radius of the heap =36 cm.

(ii) Slant height,
$\ell = \sqrt{h^2 + r^2} = \sqrt{(24)^3 + (36)^2} = \sqrt{1872} = 12\sqrt{13}$ cm.

Ex.13 An exhibition tent is in the form of a cylinder surmounted by a cone. The height of the tent above the ground is 85 m and the height of the cylindrical part is 50 m. If the diameter of the base is 168 m, find the quantity of canvas required to make the tent. Allow 20% extra for folds and for stitching. Give your answer to the nearest m^2.



Sol. Radius of the tent, r = $\left(\frac{168}{2}\right)$ m = 84 m.

Height of the tent = 85 m.

Height of the cylindrical part, H = 50 m.

Height of the conical part, h = (85 – 50) m = 35 m.

Slant height of the conical part, $\ell = \sqrt{h^2 + r^2} = \sqrt{(35)^2 + (84)^2} = \sqrt{8281}$ m = 91 m.

Quantity of canvas required

= Curved surface area of the tent

= Curved surface area of the cylindrical part

+ Curved surface area of the conical part

= $2\pi rH + \pi r\ell = \pi r(2H + \ell)$

$= \left[\frac{22}{7} \times 84(2 \times 50 + 91)\right] m^2$ = (22 × 12 × 191) m^2 = 50424 m^2.

Area of canvas required for folds and stitching = (20% of 50424) m^2 = $\left(\frac{20}{100} \times 50424\right)$ m^2 = 10084.80 m^2.

∴ Total quantity of canvas required to make the tent

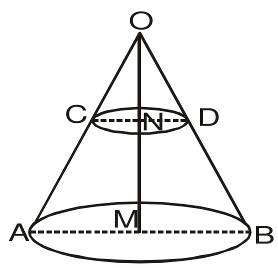
= (50424 + 10084.80) m^2 = 60508.80 m^2 = 60509 m^2. (to the nearest m^2)

Ex.14 The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to its base. If its volume be $\frac{1}{27}$ of the volume of the given cone, at what height, above the base is the section cut?

Sol. Let OAB be the given cone of height, H = 30 cm and base radius R cm. Let this cone be cut by the plane CND to obtain the cone OCD with height h cm and base radius r cm.

Then, ΔOND ~ ΔOMB.

So, $\frac{ND}{MB} = \frac{ON}{OM} \Rightarrow \frac{r}{R} = \frac{h}{30}$ ...(i)



Volume of cone OCD = $\frac{1}{27}$ × Volume of cone OAB

$\Rightarrow \quad \frac{1}{3}\pi r^2 h = \frac{1}{27} \times \frac{1}{3}\pi R^2 \times 30$

$\Rightarrow \quad \left(\frac{r}{R}\right)^2 = \frac{10}{9h} \Rightarrow \left(\frac{h}{30}\right)^2 = \frac{10}{9h}$ [From (i)]

$\Rightarrow \quad 9h^3 = 9000 \Rightarrow h^3 = 1000 \Rightarrow h = 10.$

∴ Height of the cone OCD = 10 cm.

Hence, the section is cut at the height of (30 – 10) cm, i.e., 20 cm from the base.

Ex.15 From a solid cylinder of height 30 cm and radius 7 cm, a conical cavity of height 24 cm and of base radius 7 cm is drilled out. Find the volume and the total surface of the remaining solid.

Sol. Radius, r = 7 cm.

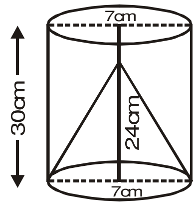
Height of the cylinder, H = 30 cm.

Height of the cone, h = 24 cm.

Slant height of the cone, $\ell = \sqrt{h^2 + r^2} = \sqrt{(24)^2 + (7)^2} = \sqrt{625} = 25$ cm

(i) Volume of the remaining solid

= (Volume of the cylinder) – (Volume of the cone)



$= \pi r^2 H - \frac{1}{3}\pi r^2 h = \pi r^2\left(H - \frac{h}{3}\right)$

$= \left[\frac{22}{7} \times 7 \times 7 \times \left(30 - \frac{24}{3}\right)\right] \text{cm}^3 = \left[\frac{22}{7} \times 7 \times 7 \times 22\right] \text{cm}^3$

$= (22 \times 7 \times 22)\ \text{cm}^3 = 3388\ \text{cm}^3$.

(ii) Total surface area of the remaining solid

= Curved surface area of cylinder + Curved surface area of cone

+ Area of (upper) circular base of cylinder

$= 2\pi rH + \pi r\ell + \pi r^2 = \pi r(2H + \ell + r) = \left[\frac{22}{7} \times 7 \times (60 + 25 + 7)\right] \text{cm}^2 = (22 \times 92)\ \text{cm}^2 = 2024\ \text{cm}^2$

Ex.16 A solid consisting of a right circular cone, standing on a hemisphere, is placed upright, in a right circular cylinder, full of water, and touches the bottom. Find the volume of water left in the cylinder, having given that the radius of the cylinder is 3 cm and its height is 6 cm; the radius of the hemisphere is 2 cm and the height of the cone is 4 cm. Give your answer to the nearest cm^3 (Take $\pi = 22/7$)

Sol. Radius of the cylinder = 3 cm and its height = 6 cm.

Volume of water in the cylinder, when full $= \left[\pi \times (3)^2 \times 6\right] \text{cm}^3 = (54\pi)\ \text{cm}^3$.

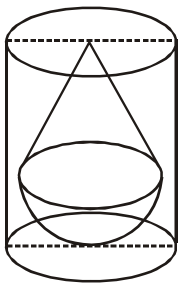
Volume of solid consisting of cone and hemisphere = (Volume of hemi-sphere) + (Volume of cone)

$= \left[\frac{2}{3}\pi \times (2)^3 + \frac{1}{3}\pi \times (2)^2 \times 4\right] \text{cm}^3 = \left(\frac{32\pi}{3}\right) \text{cm}^3$.

Volume of water displaced from cylinder

= Volume of solid consisting of cone and hemisphere

$= \left(\frac{32\pi}{3}\right) \text{cm}^3$



Volume of water left in the cylinder after placing the solid into it

$= \left(54\pi - \frac{32\pi}{3}\right) \text{cm}^3 = \left(\frac{130\pi}{3}\right) \text{cm}^3$

$= \left(\frac{130}{3} \times \frac{22}{7}\right) \text{cm}^3 = 136.19\ \text{cm}^3$.

Hence, the volume of water left in the cylinder to the nearest cm^3 is 136 cm^3.

Ex.17 The given figure shows the cross-section of an ice-cream cone consisting of a cone surmounted by a hemisphere. The radius of the hemisphere is 3.5 cm and the height of the cone is 10.5 cm. The outer shell ABCDFE is shaded and is not filled with ice-cream. AE = DC = 0.5 cm, AB||EF and BC||FD. Calculate:

(i) the volume of the ice-cream in the cone (the unshaded portion including the hemisphere) in cm^3;

(ii) the volume of the outer shell (the shaded portion) in cm^3. Give your answer to the nearest cm^3.

Sol. Radius of hemisphere, R = AG = 3.5 cm.

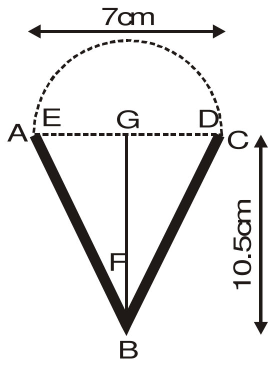
External radius of conical shell, R = AG = 3.5 cm.

Internal radius of conical shell, r = EG

= (AG – AE) = (3.5 – 0.5) cm = 3 cm

External height of conical shell, H = BG = 10.5cm.

Now, $\Delta ABG \sim \Delta EFG$.



$$\therefore \quad \frac{FG}{BG} = \frac{EG}{AG} \Rightarrow \frac{FG}{10.5} = \frac{3}{3.5} \Rightarrow FG = 9 \text{ cm.}$$

So, internal height of conical shell, h = FG = 9 cm.

(i) Volume of ice-cream

= Volume of hemisphere + Internal volume of conical shell

$$= \frac{2}{3}\pi R^3 + \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(2R^3 + r^2 h)$$

$$= \left[\frac{1}{3} \times \frac{22}{7} \times \{2 \times (3.5)^3 + (3)^2 \times 9\}\right] cm^3$$

$$= \left[\frac{1}{3} \times \frac{22}{7} \times \left(\frac{343}{4} + 81\right)\right] cm^3$$

$$= \left(\frac{1}{3} \times \frac{22}{7} \times \frac{667}{4}\right) cm^3 = \left(\frac{7337}{42}\right) cm^3$$

= 174.69 cm^3 = 175 cm^3. (to the nearest cm^3)

(ii) Volume of the shell = External volume of cone – Internal volume of cone

$$= \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(R^2 H - r^2 h)$$

$$= \frac{1}{3}\pi\left[(3.5)^2 \times 10.5 - (3)^2 \times 9\right] cm^3$$

$$= \frac{1}{3}\pi\left[\left(\frac{7}{2}\right)^2 \times \left(\frac{21}{2}\right) - (9 \times 9)\right] cm^3$$

$$= \left[\frac{1}{3} \times \frac{22}{7} \times \left(\frac{1029}{8} - 81\right)\right] cm^3$$

$$= \left(\frac{1}{3} \times \frac{22}{7} \times \frac{381}{8}\right) cm^3 = \left(\frac{1397}{28}\right) cm^3$$

= 49.89 cm^3

= 50 cm^3 (to the nearest cm^3)

Ex.18 A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The height and radius of the cylindrical part are 13 cm and 5 cm respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part. Calculate the surface area of the toy if the height of the conical part is 12 cm.

Sol. The toy is in the shape shown below :

Radius of the hemispherical part = 5 cm.

$\therefore$ Curved surface area of the Hemispherical part

$= 2\pi r^2 = [2\pi \times (5)^2]\ cm^2 = (50\pi)\ cm^2$.

Cylindrical part has radius = 5 cm and height

= 13 cm. i.e. h

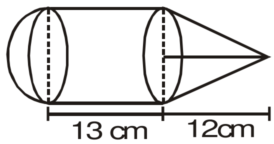
$\therefore$ Curved surface area of the cylindrical part

$= 2\pi rh = (2\pi \times 5 \times 13)\ cm^2 = (130\pi)\ cm^2$.

Conical part has radius =5 cm and height =12 cm.

$\therefore$ Its slant height

$= \sqrt{5^2 + (12)^2} = \sqrt{25 + 144} = \sqrt{169} = 13$ cm.



$\therefore$ Curved surface area of the conical part= $\pi r\ell$

$= (\pi \times 5 \times 13)\ cm^2 = (65\pi)\ cm^2$

Hence, the surface area of the toy

$= (50\pi + 130\pi + 65\pi)\ cm^2 = (245\pi)\ cm^2. \quad = \left(245 \times \frac{22}{7}\right) cm^2 = 770\ cm^2$.

Also, volume of the toy $= \left(\frac{2}{3}\pi r^3 + \pi r^2 h + \frac{1}{3}\pi r^2 H\right)\ cm^3$

$= \left(\frac{250\pi}{3} + 325\pi + 100\pi\right)\ cm^3$

$= \left(\frac{1525\pi}{3}\right) cm^3$

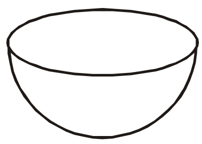
$= \left(\frac{1525}{3} \times \frac{22}{7}\right) cm^3 = 1597.6\ cm^3$

Ex.19 The outer and inner diameters of a hemispherical bowl are 17 cm and 15 cm respectively. Find the cost of polishing it all over at 25 paise per cm^2. (Take $\pi = 22/7$).

Sol. Outer radius (R) $= \frac{17}{2}$ cm, Inner radius (r) $= \frac{15}{2}$ cm.

Area of outer surface $= 2\pi R^2 = \left[2\pi \times \left(\frac{17}{2}\right)^2\right] cm^2 = \left(\frac{289\pi}{2}\right) cm^2$.

Area of inner surface $= 2\pi r^2$



$= \left[2\pi \times \left(\frac{15}{2}\right)^2\right] cm^2 = \left(\frac{225\pi}{2}\right) cm^2$.

Area of the ring at the top $= \pi (R^2 - r^2) = \pi [(8.5)^2 - (7.5)^2]\ cm^2 = (16\pi)\ cm^2$.

$\therefore$ Total area to be polished

$= \left(\frac{289\pi}{2} + \frac{225\pi}{2} + 16\pi\right) cm^2$

$= (273\pi)\ cm^2 = \left(273 \times \frac{22}{7}\right) cm^2 = 858\ cm^2$.

$\therefore$ Cost of polishing the bowl = Rs $\left(\frac{858 \times 25}{100}\right)$ = Rs. 214.50.

Ex.20 A conical vessel of radius 6 cm and height 8 cm is completely filled with water. A sphere is lowered into the water and its size is such that when it touches the sides, it is just immersed. What fraction of water overflows?

Sol. Radius of the conical vessel, R = AC = 6 cm.

Height of the conical vessel, H = OC = 8 cm.

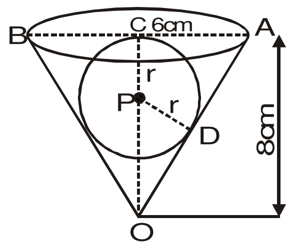
Let the radius of the sphere be r.

Then, PC = PD = r.

Now, AC = AD = 6 cm.

[$\because$ lengths of two tangents from an external point to a circle are equal]

OA = $\sqrt{OC^2 + AC^2} = \sqrt{8^2 + 6^2} = \sqrt{100}$ = 10 cm.



OD = (OA – AD) = (10 – 6) cm = 4 cm.

OP = (OC – PC) = (8 – r).

In right angled ΔODP, we have :

$$OP^2 = OD^2 + PD^2$$

$\Rightarrow (8 - r)^2 = 4^2 + r^2 \Rightarrow 64 - 16r + r^2 = 16 + r^2$

$\Rightarrow 16r = 48 \Rightarrow r = \frac{48}{16}$ = 3cm.

Volume of water overflow = Volume of sphere = $\frac{4}{3}\pi r^3 = \left[\frac{4}{3}\pi \times (3)^3\right]$ cm^3 = (36π) cm^3.

Volume of water in the cone before immersing the sphere

= Volume of cone = $\frac{1}{3}\pi r^2 h = \left(\frac{1}{3}\pi \times (6)^2 \times 8\right)$ cm^3 = (96π) cm^3.

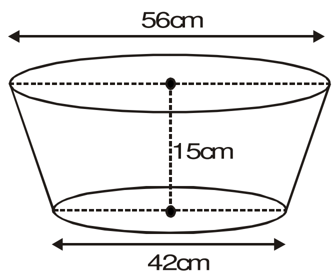
$\therefore$ Fraction of water overflow = $\frac{\text{Volume of water overflow}}{\text{Original volume of water}} = \frac{(36\pi)}{(96\pi)} = \frac{3}{8}$

Ex.21 A bucket is in the form of a frustum of a cone, its depth is 15 cm and the diameters of the top and the bottom are 56 cm and 42 cm respectively. Find how many litres of water can the bucket hold ? (Take π = 22/7)

Sol. R = 28 cm

r = 21 cm

h = 15 cm



Capacity of the bucket = $\frac{1}{3}\pi h \{R^2 + r^2 + R r\}$

$= \frac{1}{3} \times \frac{22}{7} \times 15 \times \{(28)^2 + (21)^2 + (28)(21)\}$ cm^3

$= \frac{22}{7} \times 5 \times \{784 + 441 + 588\}$ cm^3

$= \frac{22}{7} \times 5 \times 1813$ cm3

= 22 × 5 × 259 cm^3

= 28490 cm^3 = $\frac{28490}{1000}$ litres = 28·49 litres

Ex.22 A bucket of height 16 cm and made up of metal sheet is in the form of frustum of a right circular cone with radii of its lower and upper ends as 3 cm and 15 cm respectively. Calculate:

(i) the height of the cone of which the bucket is a part.

(ii) the volume of water which can be filled in the bucket.

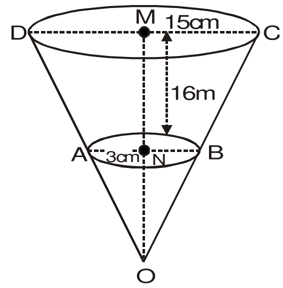
(iii) the slant height of the bucket.

(iv) the area of the metal sheet required to make the bucket.

Sol. Let ABCD be the bucket which is frustum of a cone with vertex 0 (as shown in figure). Let ON = x cm

$\Delta ONB - \Delta OMC$

$\frac{x}{16+x} = \frac{3}{15}$ $\left\{\because \frac{ON}{OM} = \frac{NB}{MC}\right\}$



$\Rightarrow$ $\frac{x}{16+x} = \frac{1}{5}$ $\Rightarrow$ $5x = 16 + x1$

$\Rightarrow$ $4x = 16$ $\Rightarrow$ $x = 4$

$\therefore$ ON = 4 cm and OM = 4 + 16 = 20 cm

$\therefore$ the height of the cone = 20 cm

volume of the bucket = $\frac{1}{3}\pi (15)^2 \times 20 - \frac{1}{3}\pi (3)^2 \times 4$ cm^3

{i.e., Volume of the large cone - Volume of the small cone}

$= \frac{1}{3}\pi [225 \times 20 - 36]$ cm^3

$= \pi [75 \times 20 - 12]$ cm^3

$= 1488\,\pi$ cm^3

Slant height of cone of radius 15 cm

$= \sqrt{(15)^2 + (20)^2}$ cm $= \sqrt{625}$ cm = 25 cm

Slant height of cone of radius 3 cm

$= \sqrt{(4)^2 + (3)^2}$ = 5 cm

$\therefore$ Slant height of bucket = (25 – 5) cm = 20 cm, i.e, ℓ = 20 cm

$\therefore$ The area of the metal sheet

$= \pi \ell (R + r) + \pi r^2$

$= \pi \times 20 \times (15 + 3) + \pi \times (3)^2$ cm^2

$= 360\,\pi + 9\,\pi$ cm^2

$= 369\,\pi$ cm^2

Note. The area of the metal sheet used = C.S. of larger cone – C.S. of smaller cone + Area of the base of the bucket

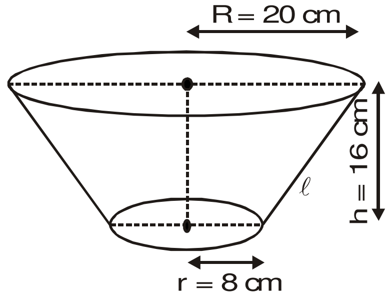
$= [\pi \times 25 \times 15 - \pi \times 5 \times 3 + \pi \times (3)^2]$ cm^2 = $[375\,\pi - 15\,\pi + 9\,\pi]$ cm^2

$= 369\,\pi$ cm^2

Ex.23 A container made up of a metal sheet is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of the milk which can completely fill the container at the rate of Rs. 15 per litre and the cost of the metal sheet used, if it costs Rs. 5 per 100 cm^2. (Take π 3·14) **[NCERT]**

Sol. R = 20 cm, r = 8cm, h = 16 cm

$\ell = \sqrt{h^2 + (R - r)^2} = \sqrt{256 + 144}$ cm = 20 cm



Volume of container = $\frac{1}{3}\pi h \{R^2 + r^2 + R\,r\}$

$= \frac{1}{3} \times (3.14) \times 16 \{400 + 64 + 160\}$ cm^3

$= 3.14 \times \frac{16}{3} \{624\}$ cm^3

$= 3.14 \times 16 \times 208$ cm^3

$= 10449{\cdot}92$ cm^3

Therefore, the quantity of milk in the container = $\frac{10449.92}{1000}$ litres = 10·45 litres

Cost of milk at the rate of Rs. 15 per litre = Rs. {10·45 × 15} = Rs.156·75

Surface area of the metal sheet used to make the container

$= \pi \ell (R + r) + \pi r^2 = \pi \{\ell (R + r) + r^2\}$

$= (3{\cdot}14) \times \{20 \times 28 + 64\}$ cm^2

$= (3{\cdot}14) \times 624$ cm^2 $= 1959{\cdot}36$ cm^2

Therefore, the cost of the metal sheet at rate of Rs. 5 per 100 cm^2

$=$ Rs. $\frac{1959.36 \times 5}{100}$ = Rs. 97·97 approx.

Ex.24 The height of a cone is 40 cm. A small cone is cut off at the top by a plane parallel to the base. If the volume of the small cone be $\frac{1}{64}$ of the volume of the given cone, at what height above the base is the section made ?

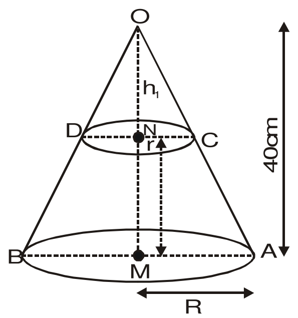
Sol. Let R be the radius of the given cone, r the radius of the small cone, h be the height of the frustum and h_1 be the height of the small cone.

In figure 13.49, ΔONC and ΔOMA are similar (ΔONC ~ ΔOMA)

$\therefore \quad \frac{ON}{OM} = \frac{NC}{MA} \quad \Rightarrow \quad \frac{h_1}{40} = \frac{r}{R}$

$\Rightarrow \quad h_1 = \left(\frac{r}{R}\right)40$ ...(i)

We are given that $\frac{\text{Volume of small cone}}{\text{Volume of given cone}} = \frac{1}{64}$



$\Rightarrow \quad \frac{\frac{1}{3}\pi r^2 \times h_1}{\frac{1}{3}\pi R^2 \times 40} = \frac{1}{64}$

$\Rightarrow \quad \frac{r_2}{R_2} \times \frac{1}{40} \times \left\{\left(\frac{r}{R}\right)40\right\} = \frac{1}{64}$ (By i)

$\Rightarrow \quad \left(\frac{r}{R}\right)^3 = \frac{1}{64} = \left(\frac{1}{4}\right)^3 \Rightarrow \frac{r}{R} = \frac{1}{4}$ ...(ii)

From (i) and (ii) $\quad h_1 = \frac{1}{4} \times 40 = 10$ cm

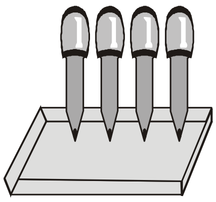
Therefore, $\quad h = 40 - h_1 = (40 - 10)$ cm

$\Rightarrow \quad h = 30$ cm Hence, the section is made at a height of 30 cm above the base of the cone.

EXERCISE – I UNSOLVED PROBLEMS

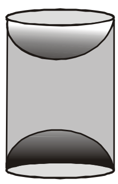
Q.1 A tent is in the shape of a cylinder surmounted by a conical top. If the height and the diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs 500 per m^2.

Q.2 A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm b y 3.5 cm. The radius of each of the depression is 0.5 cm and the depth is 1.4 cm. Find the volume of the wood in the entire stand.

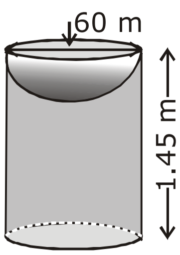


Q.3 From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.

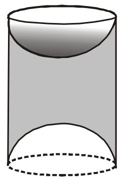
Q.4 A wooden article was made by scooping out a hemisphere from each end of a solid cylinder (as shown in the adjoining figure). If the height of the cyclinder is 10 cm and its base is of radius 3.5 cm, find the total surface area of the article.



Q.5 Rashmi made a bird-bath for her garden in the shape of a cylinder with a hemispherical depression at one end (as shown in the adjoining figure). The height of the cylinder is 1.45 m and its diameter is 60 cm. Find the total surface area of the bird-bath.



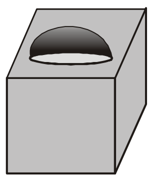
Q.6 A juice seller was serving his customers using glasses as shown in the adjoining figure. The inner diameter of the cylinderical glass was 5 cm, but the bottom of the glass had a hemispherical portion raised which reduceds the capacity of the glass. If the height of a glass was 10 cm, find the apparent capacity of the glass and its actual capacity. (Use π = 3.14)



Q.7 (i) A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open , is 5 cm. It is filled with water upto the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped into the vessel.

(ii) A vessel is in the form of an inverted cone. Its height is 11 cm and the radius of its top, which is open, is 2.5 cm. It is filled with water upto the rim. When some lead shots, each of which is a sphere of radius 0.25 cm, are dropped into the vessel $\frac{2}{5}$ of the water flows out. Find the number of lead shots dropped into the vessel.

Q.8 The adjoining figure shows a decorative block, which is made of two solids ––a cube and a hemisphere. The base of the block is a cube with dege 5 cm, and hemisphere fixed on the top has a diameter of 4.2 cm. Find the total volume and the total surface area of the block.



Q.9 A cuboidal block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the sphere can have ? Also find the surface area of the solid.

Q.10 A hemispherecal depression is cut out from one face of a cuboidal block such that the diameter *l* (cm) of the hemisphere is equal to the edge of the cube. Find the volume and the surface area of the remaining solid.

Q.11 A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

Q.12 The adjoining figure shows a medicine capsule, which is in the shape of a cylinder with two hemispheres struck to each of its ends. If the length of the capsule is 14 mm and the diameter of the capsule is 5 mm, find its surface area.

Q.13 A *gulabjamun,* contains sugar syrup approximately 30% of its volume. Find how much syrup would be found in 45 *gulabjamuns,* each shaped like a cylinder with two hemispherical ends with lengths 5 cm and diameter 2.8 cm.

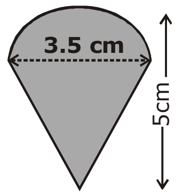
Q.14 A spherical glass vessel has cylindrical neck 8 cm long, 2 cm in diameter, the diameter of the spherical part is 8.5 cm. Find the amount of water it can hold, taking the above as inside measurements. (Take $\pi = 3.14$)

Q.15 A student was asked to make a model shaped like a cylinder with two cones attached at its ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model.

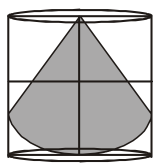
Q.16 The internal and external diameters of a hollow hemispherical vessel are 14cm and 21cm respectively. The cost of silver plating of 1 cm^2 surface is Rs 0.60. Find the total cost of silver plating the vessel all over.

Q.17 A solid is in the shape of a cone surmounted on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid. (Leave your answer in π)

Q.18 The adjoining figure shows a playing top (*lattu*). The top is shaped like a cone surmounted by a hemisphere. The entire top is 5 cm in height and the diameter of the top is 3.5 cm. Find its surface area.



Q.19 A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy. If a right circular cylinder circumscribes the toy (shown in the adjoining figure), find the difference of the volumes of the cylinder and the toy. (Take $\pi = 3.14$)



Q.20 A solid consisting of a right circular cone of height 120 cm and radius 60 cm surmounted on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of the water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

Q.21 The volume of a cone is the same as that of the cylinder whose height is 9 cm and diameter 40 cm. Find the radius of the base of the cone if its height is 108 cm.

Q.22 A right circular cone of height 20 cm and base diameter 30 cm is cast into smaller cones of equal sizes with base radius 10 cm and height 9 cm. Find how many cones are made.

Q.23 A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Q.24 A girl fills a cylindrical bucket 32 cm in height and 18 cm in radius with sand. She empties the bucket on the ground and makes a conical heap of the sand. If the height of the conical heap is 24 cm, find

(i) the radius and

(ii) the slant height of the heap.

Leave your answer in square root form.

Q.25 The entire surface of a solid cone of base radius 3 cm and height 4 cm is equal to the entire surface of a solid right circular cylinder of diameter 4 cm. Find the ratio of

(i) their curved surfaces.

(ii) their volumes.

Q.26 Two spheres of the same metal weigh 1 kg and 7 kg. The radius of the smaller sphere is 3 cm. The two spheres are melted to form a single big sphere. Find the diameter of this big sphere.

Q.27 The surface area of a solid metallic sphere is 1256 cm^2. It is melted and recast into solid right circular cones of radius 2.5 cm and height 8 cm. Calculate

(i) the radius of the solid sphere.

(ii) the number of cones recast.

(Take π = 3.14)

Q.28 A spherical copper shell of external diameter 18 cm is melted and recasted into a solid cone of base radius 14 cm and height $4\frac{3}{7}$ cm. Find the inner diameter of the shell.

ANSWER KEY

1. 44 m^2 , Rs 22000

2. 523.53 cm^3 **3.** 18 cm^2

4. 374 cm^2 **5.** 3.3 m^2

6. 196.25 cm^2 ; 163.54 cm^3

7. (i) 100 (ii) 440

8. 144.404 cm^3; 163.86 cm^2

9. 7 cm; 332.5 cm^2

10. $\frac{l^3}{12}(12-\pi)$ cm^3; $\frac{l^2}{4}$ $(\pi + 24)$ cm^2

11. 572 cm^2

12. 220 mm^2 **13.** 338.18 cm^3

14. 346.51 cm^3 **15.** 66 cm^3

16. Rs. 716.10 **17.** π cm^3

18. 39.6 cm^2 (approx)

19. 25.12 cm^3; 25.12 cm^3

20. 1.131 m^3 **21.** 10 cm

22. 5 **23.** 2.744 cm

24. (i) 36 cm (ii) $12\sqrt{13}$ cm

25. (i) 15 : 16 (ii) 3 : 4

26. 12 cm

27. (i) 10 cm (ii) 80

28. 16 cm

EXERCISE – II BOARD PROBLEMS

Q.1 The surface area of a sphere is 616 cm^2. Find its radius. **[Foreign – 2008]**

Q.2 A cylinder and a cone are of same base radius and of same height. Find the ratio of the volume of cylinder to that of the cone. **[Delhi – 2009]**

Q.3 The slant height of the frustum of a cone is 5 cm. If the difference between the radii of its two circular ends is 4 cm, write the height of the frustum. **[AI – 2010]**

Q.4 A solid metallic sphere of diameter 21 cm is melted and recasted into a number of smaller cones, each of diameter 7 cm and height 3 cm. Find number of cones so formed. **[Delhi – 2004]**

Q.5 A solid metallic sphere of diameter 28 cm is melted and recasted into a number of smaller cones, each of diameter $4\frac{2}{3}$ cm and height 3cm. Find the number of cones so formed. **[Delhi – 2004]**

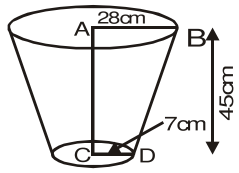
Q.6 A hemispherical bowl of internal diameter 30 cm contains some liquid. This liquid is to be filled into cylindrical shaped bottles each of diameter 5 cm and height 6 cm. Find the number of bottles necessary to empty the bowl. **[AI – 2004]**

Q.7 Solid spheres of diameter 6 cm are dropped into a cylindrical beaker containing some water and are fully submerged. If the diameter of the beaker is 18 cm and the water rises by 40 cm, find the number of solid spheres dropped in the water. **[Foreign – 2004]**

Q.8 A toy is in the form of a cone mounted on a hemisphere of common base radius 7 cm. The total height of the toy is 31 cm. Find the total surface area of the toy. [use π = 22/7] **[Delhi – 2007]**

Q.9 A toy is in the form of a cone mounted on a hemisphere with same radius. The diameter of the base of the conical portion is 7 cm and total height of the toy is 14.5 cm. Find the volume of the toy.[use π = 22/7] **[AI – 2007]**

Q.10 If the radii of the circular ends of a bucket, 45 cm high are 28 cm and 7 cm (as shown in given fig.), find the capacity of the bucket, curved surface area & total surface area. **[AI – 2004]**



Q.11 A hollow cone is cut by a plane parallel to the base and the upper portion is removed. If the curved surface of the remainder is 8/9th of the curved surface of the whole cone, find the ratio of the line segments into which the cone's altitude is divided by the plane.

Q.12 A well, of diameter 3m, is dug 14 m deep. The earth taken out of it has been spread evenly all around it to a width of 4m, to form an embankment. Find the height of the embankment. [use π = 22/7] **[AI – 2004C]**

Q.13 The rain water from a roof 22 m × 20 m drains into a cylindrical vessel having diameter of base 2 m and height 3.5 m. If the vessel is just full, find the rainfall in cm. **[Delhi – 2006]**

Q.14 Water flows at the rate of 10 m per minute through a pipe having its diameter as 5 mm? How much time will it take to fill a conical vessel whose diameter of base is 40 cm and depth 24 cm? **[Foreign –2006]**

Q.15 A sphere, of diameter 12 cm, is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level in the cylindrical vessel rises by $3\frac{5}{9}$ cm . Find the diameter of the cylindrical vessel.

OR

A solid right circular cone of diameter 14 cm and height 8 cm is melted to form a hollow sphere. If the external diameter of the sphere is 10 cm, find the internal diameter of the sphere. **[Delhi – 2007]**

Q.16 A hemispherical bowl of internal diameter 36 cm is full of some liquid. This liquid is to be filled in cylindrical bottles of radius 3 cm and height 6 cm. Find the number of bottles needed to empty the bowl.

OR

Water flows out through a circular pipe whose internal radius is 1 cm, at the rate of 80 cm/second into an empty cylindrical tank, the radius of whose base is 40 cm. By how much will the level of water rise in the tank in half an hour. **[AI – 2007]**

Q.17 A gulab jamun, when ready for eating, contains sugar syrup of about 30% of its volume. Find approximately how much syrup would be found in 45 such gulab jamuns, each shaped like a cylinder with two hemispherical ends, if the complete length of each of them is 5 cm and it's diameter is 2.8 cm.

OR

A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of icecream. This ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with icecream. **[Delhi – 2008]**

Q.18 A bucket made up of a metal sheet is in the form of a frustum of a cone of height 16 cm with diameters of it's lower and upper ends as 16 cm and 40 cm respectively. Find the volume of the bucket. Also find the cost of the bucket if the cost of metal sheet used is Rs. 20 per 100 cm^2 [use π = 3.14]

OR

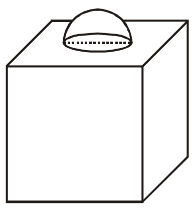
A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in his field which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 6 km/hr, in how much time will the tank be filled? **[Delhi – 2008]**

Q.19 A tent consists of a frustum of a cone, surmounted by a cone. If the diameter of the upper and lower circular ends of the frustum be 14 m and 26 m respectively, the height of the frustum be 8 m and the slant height of the surmounted conical portion be 12 m, find the area of canvas required to make the tent. (Assume that the radii of the upper circular end of the frustum and the base of surmounted conical portion are equal). **[AI – 2008]**

Q.20 If the radii of the circular ends of a conical bucket, which is 16 cm high, are 20 cm and 8 cm, find the capacity and total surface area of the bucket. [use π = 22/7] **[Foreign – 2008]**

Q.21 From a solid cylinder whose height is 8 cm and radius 6 cm, a conical cavity of height 8 cm and of base radius 6 cm, is hollowed out. Find the volume of the remaining solid correct to two places of decimals. Also, find the total surface of the remaining solid. [Take $\pi = 3.1416$] **[Delhi – 2009]**

Q.22 In figure, a decorative block which is made of two solids – a cube and a hemisphere. The base of the block is a cube with edge 5 cm and the hemisphere, fixed on the top, has a diameter of 4.2 cm. Find the total surface area of the block. [Take $\pi = 22/7$] **[AI – 2009]**



Q.23 A spherical copper shell, of external diameter 18 cm, is melted and recast into a solid cone of base radius 14 cm and height $4\frac{3}{7}$ cm. Find the inner diameter of the shell.

OR

A bucket is in the form of a frustum of a cone with a capacity of 12308.8 cm^3. The radii of the top and bottom circular ends of the bucket are 20 cm and 12 cm respectively. Find the height of the bucket and also the area of metal sheet used in making it. [Use $\pi = 3.14$]

ANSWER KEY

1.	7 cm
2.	3 : 1
3.	3 cm
4.	126
5.	672
6.	60
7.	90
8.	858 cm^2
9.	231 cm^3
10.	48510 cm^3, 5462.6 cm^2, 8080.6 cm^2
11.	1 : 2
12.	1.125 m
13.	2.5 cm
14.	51.2 min
15.	18 cm OR 6 cm
16.	72 OR 90 cm
17.	338.184 cm^3 OR 10 cones
18.	10.45 ltrs, Rs. 391.87 OR 50 minutes
19.	(284π) m^2
20.	$\frac{73216}{7} cm^3, \frac{13728}{7} cm^2$
21.	150.79 cm^3, 259.55 cm^2
22.	163.86 cm^2
23.	16 cm OR 15 cm ; 2160.32 cm^2

EXERCISE – III MULTIPLE CHOICE QUESTIONS

Q.1 A solid sphere of radius r is melted and cast into the shape of a solid cone of height r. Radius of the base of the cone is :

(A) r (B) 2r
(C) 3r (D) 4r

Q.2 A solid sphere of diameter 24 cm is melted and recast into spherical balls of 2 cm diameter. The number of such balls made from the material, is

(A) 1728 (B) 864
(C) 576 (D) 3456

Q.3 A metallic sphere of diameter 21 cm is melted and then recast into small cones each of diameter 7 cm and height 3 cm. The number of cones thus made is

(A) 21 (B) 63
(C) 126 (D) 130

Q.4 The number of solid spheres, each of radius 3 cm that could be moulded to form a metallic cylinder of height 45 cm and base radius 2 cm is

(A) 3 (B) 4
(C) 6 (D) 5

Q.5 If a cone is cut into the parts by horizontal plane passing through the mid-point of its axis, the ratio of the volumes of the upper part and the cone is

(A) 1 : 2 (B) 1 : 4
(C) 1 : 6 (D) 1 : 8

Q.6 The material of a cone is converted into the shape of cylinder of equal radius. If the height of the cylinder is 5 cm, then height of the cone is

(A) 10 cm (B) 15 cm
(C) 18 cm (D) 24 cm

Q.7 If the radii of the circular ends of a bucket of height 40 cm are of lengths 35 cm and 14 cm, the n the volume of the bucket is

(A) 60060 cu cm (B) 70040 cu cm
(C) 80080 cu cm (D) 80160 cu cm

Q.8 A sphere of diameter 12 cm is dropped into a clyindrical vessel partly filled with water. The diameter of the vessel is 16 cm. If the sphere is completely submerged, then the water level rises by

(A) 2 cm (B) 3 cm
(C) 4 cm (D) 4.5 cm

Q.9 The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to its base. If its volume be $\frac{1}{27}$ of the volume of the given cone, then the height above the base at which the section is made, is

(A) 15 cm (B) 20 cm
(C) 25 cm (D) 10 cm

Q.10 A solid is hemispherical at the bottom and conical above it. If the surface areas of the two parts are equal, then the ratio of its radius and the height of its conical part is :

(A) $1 : \sqrt{3}$ (B) 1 : 3
(C) $\sqrt{3} : 1$ (D) 1 : 1

Q.11 The base radii of a cone and a cylinder are equal. If their volumes are also equal, then the ratio of height of cone to height of cylinder is

(A) 2 : 1 (B) 1 : 2
(C) 1 : 3 (D) 3 : 1

Q.12 The base radii of a cone and a clyinder are equal. If their curved surface areas are also equal, then the ratio of the slant height of the cone to the height of the cylinder is

(A) 2 : 1 (B) 1 : 2
(C) 1 : 3 (D) 3 : 1

Q.13 If two cones have their heights in the ratio 1 : 3 and radii in the ratio 3 :1, then the ratio of their volumes is

(A) 1 : 3 (B) 3 : 1
(C) 3 : 2 (D) 2 : 3

Q.14 If the height and radius of a cone are doubled, then the volume of the cone becomes

(A) 2 times (B) 4 times
(C) 8 times (D) 12 times

Q.15 If the perimeter of one face of a cube is 20 cm, then its surface area is

(A) 120 cm^2 (B) 150 cm^2

(C) 125 cm^2 (D) 400 cm^2

Q.16 If the diameter of a right circular cylinder is 10 cm and height is 4 cm, then its total surface area is

(A) 4 π cm^2 (B) 65 π cm^2

(C) 90 π cm^2 (D) 120 π cm^2

Q.17 The volume of a right circular cone of height 8 cm and diameter of base 6 cm is

(A) 12 π cm^3 (B) 24 π cm^3

(C) 48 π cm^3 (D) 96 π cm^3

Q.18 If a cone of height 8 cm has base diameter 12 cm, then its curved surface area is

(A) 36 π cm^2 (B) 48 π cm^2

(C) 60 π cm^2 (D) 72 π cm^2

Q.19 If the ratio of surface areas of two spheres is 4 : 9, then the ratio of their volumes is

(A) 8 : 27 (B) 27 : 8

(C) 2 : 3 (D) 3 : 2

Q.20 If the vertical height of a cone is 8 cm and the area of its base is 156 cm^2, then its volume is

(A) 312 cm^3 (B) 468 cm^3

(C) 415 cm^3 (D) 416 cm^3

Q.21 If the base area of a cone is 51 cm^2 and its volume is 85 cm^3, then its vertical height is

(A) 3.5 cm (B) 4 cm

(C) 4.5 cm (D) 5 cm

Q.22 If the surface area of a sphere is 324 π cm^2, then its volume is

(A) 960 π cm^3 (B) 972 cm^3

(C) 729 π cm^3 (D) 546.75 cm^3

Q.23 A rectangular piece of paper of dimensions 100 cm × 44 cm is rolled along its length to form a cylinder. The volume of the cylinder so formed is

(A) 15400 cm^3 (B) 7700 cm^3

(C) 30800 cm^3 (D) 15600 cm^3

Q.24 If the radii of the circular bases of a frustum of a cone are 6 cm, 8 cm and its slant height is 5 cm, then the curved surface area of the frustum is

(A) 1100 cm^2 (B) 440 cm^2

(C) 220 cm^2 (D) 110 cm^2

Q.25 If the radii of the circular ends of a bucket 12 cm high are 15 cm and 10 cm, then its curved surface area is

(A) 180 π cm^2 (B) 240 π cm^2

(C) 300 π cm^2 (D) 325 π cm^2

Q.26 If the radii of the circular ends of a bucket 45 cm high are 28 cm and 7 cm, then the capacity of the bucket is

(A) 48150 cm^3 (B) 48510 cm^3

(C) 97020 cm^3 (D) 24255 cm^3

Q.27 If the solid of one shape is converted to another, then the volume of the new solid

(A) remains same (B) increases

(C) decreases (D) can't say

Q.28 If a solid of one shape is converted to another, then the surface area of the new solid

(A) remains same (B) increases

(C) decreases (D) can't say

Q.29 If a solid right circular cone of height 24 cm and base radius 6 cm and height 10 cm is melted and recast in the shape of a sphere then the radius of the sphere is.

(A) 4 cm (B) 6 cm

(C) 8 cm (D) 12 cm

Q.30 If a solid circular cylinder of iron whose diameter is 15 cm and height 10 cm is melted and recasted into a sphere, then the radius of the sphere is

(A) 15 cm (B) 10 cm

(C) 7.5 cm (D) 5 cm

Q.31 The number of balls of radius 1 cm that can be made from a sphere of radius 10 cm is

(A) 100 (B) 1000

(C) 10000 (D) 100000

Q.32 The number of balls of radius 2 cm that can be made from a cube of side 44 cm is
(A) 2500 (B) 2525
(C) 2541 (D) 2580

Q.33 If a solid sphere of radius 6 cm is melted and drawn into a wire of radius 0.2 cm, then the length of the wire is
(A) 72 m (B) 75 m
(C) 72 cm (D) 75 cm

Q.34 If a sphere and a cube have equal surface areas, then the ratio of the diameter of the sphere to the edge of the cube is
(A) 1 : 2 (B) 2 : 1
(C) $\sqrt{\pi} : \sqrt{6}$ (D) $\sqrt{6} : \sqrt{\pi}$

Q.35 If three solid sphere of radii 6 cm, 8 cm and 10 cm are melted to from a sphere, then the radius of the sphere so formed is
(A) 12 cm (B) 14 cm
(C) 16 cm (D) 24 cm

Q.36 The volume of the greatest sphere cut off from a circular cylindrical wood of base radius 1 cm and height 6 cm is
(A) 288 π cm^3 (B) $\frac{4}{3}\pi$ cm^3 `
(C) 6 π cm^3 (D) 4 π cm^3

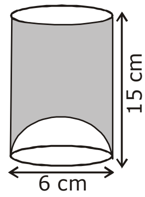
Q.37 If the radius of a sphere is increased by 50%, then its surface area is increased by
(A) 50% (B) 100%
(C) 125% (D) 150%

Q.38 If a sector of a circle of radius 6 cm and of central angle 120º is rolled up so that the two bounding radii are joined together to form a cone, then the radius of the resulting cone is
(A) 2 cm (B) 3 cm
(C) 4 cm (D) 6 cm

Q.39 The volume of the cone formed in the above question is
(A) 12 π cm^3 (B) $\frac{16}{3}\sqrt{2}\ \pi$ cm^3
(C) $\frac{8}{3}\sqrt{2}\ \pi$ cm^3 (D) $\frac{32}{3}\ \pi$ cm^3

Q.40 If cone, a hemisphere and a cylinder have equal bases and have same height ,then the ratio of their volumes is
(A) 1 : 3 : 2 (B) 2 : 3 : 1
(C) 2 : 1 : 3 (D) 1 : 2 : 3

Q.41 In the adjoining figure, the bottom of the glass has a hemispherical raised portion. If the glass is filled with orange juice, the quantity of juice which a person will get is
(A) 135 π cm^3
(B) 117 π cm^3
(C) 99 π cm^3
(D) 36 π cm^3

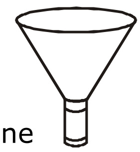


Q.42 **A cylindrical** pencil a sharpened at one edge is the combination of
(A) a cylinder and a cone
(B) a cylinder and frustum of a cone
(C) a cylinder and a hemisphere
(D) two cylinders

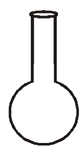
Q.43 **A shuttlecock** used for playing badminton is the combination of
(A) cylinder and a hemisphere
(B) frustum of a cone and a hemisphere
(C) a cone and a hemisphere
(D) a cylinder and a sphere


Shuttlecock

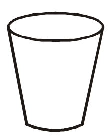
Q.44 **A funnel** is the combination of
(A) a cylinder and a cone
(B) a cylinder and a hemisphere
(C) a cylinder and frustum of a cone
(D) a cone and a hemisphere


Funnel

Q.45 **A surahi** is a combination of
(A) a sphere and a cylinder
(B) a hemisphere and a cylinder
(C) a cylinder and a cone
(D) two hemispheres


Surahi

Q.46 **The shape** of a glass (tumbler) is usually in the form of
(A) a cylinder
(B) frustum of a cone
(C) a cone
(D) a sphere


Glass

Q.47 The shape of a **gilli** in the gilli-danda game is a combination of

(A) a cone and a cylinder

(B) two cylinder

(C) two cones and a cylinder

(D) two cylinders and a cone

Gilli

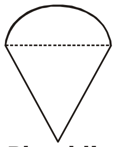
Q.48 **A plumbline** (sahul) is the combination of

(A) a hemisphere and a cone

(B) a cylinder and a cone

(C) a cylinder and frustum of a cone

(D) a cylinder and a sphere



Plumbline

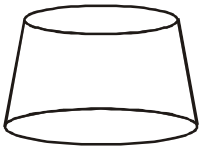
Q.49 **A cone** is cut by a plane parallel to its base and the upper part is removed. The part that is left over is called

(A) a cone

(B) a sphere

(C) a cylinder

(D) frustum of a cone



Q.50 During conversion of a solid from one shape to another, the volume of the new shape will

(A) decrease (B) increase

(C) remain unaltered (D) be doubled

Q.51 In a right circular cone, the cross-section made by a plane parallel to the base is a

(A) sphere (B) hemisphere

(C) circle (D) a semi-circle

Q.52 A solid piece of iron in the form of a cuboid of dimensions (49 cm × 33 cm × 24 cm) is moulded to form a solid sphere. The radius of the sphere is

(A) 19 cm (B) 21 cm

(C) 23 cm (D) 25 cm

Q.53 A cubical ice-cream brick of edge 22 cm is to be distributed among some children by filling ice-cream cones of radius 2 cm and height 7 cm upto its brim. How many children will get the ice-cream cones ?

(A) 163 (B) 263

(C) 363 (D)463

Q.54 A mason constructs a wall of dimensions (270 cm × 300 cm × 350 cm) with bricks, each of size (22.5 cm × 11.25 cm × 8.75 cm) and its is assumed that $\frac{1}{8}$ space is covered by the mortar. Number of bricks used to construct the wall is

(A) 11000 (B) 11100

(C) 11200 (D) 11300

Q.55 Twelve solid spheres of the same size are made by melting a solid metallic cylinder of base diameter 2 cm and height 16 cm. The diameter of each sphere is

(A) 2 cm (B) 3 cm

(C) 4 cm (D) 6 cm

Q.56 The diameters of two circular ends of a bucket are 44 cm and 24 cm and the height of the bucket is 35 cm. The capacity of the bucket is

(A) 31.7 litres (B) 32.7 litres

(C) 33.7 litres (D) 43.7 litres

Q.57 The slant height of a bucket is 45 cm and the radii of its top and bottom are 28 cm and 7 cm respectively. The curved surface area of the bucket is

(A) 4953 cm^2 (B) 4952 cm^2

(C) 4951 cm^2 (D) 4950 cm^2

Q.58 The volumes of two spheres are in the ratio 64 : 27. The ratio of their surface areas is

(A) 9 : 16 (B) 16 : 9

(C) 3 : 4 (D) 4 : 3

Q.59 A hollow cube of internal edge 22 cm is filled with spherical marbles of diameter 0.5 cm and $\frac{1}{8}$ space of the cube remains unfilled. Number of marbles required are

(A) 142296 (B) 142396

(C) 142496 (D) 142596

Q.60 A metallic spherical shell of internal and external diameters 4 cm and 8 cm respectively, is melted and recast into the form of a cone of base diameter 8 cm. The height of the cone is

(A) 12 cm (B) 14 cm

(C) 15 cm (D) 8 cm

Q.61 A medicine - capsule is in the shape of a cylinder of diameter 0.5 cm with two hemisphere stuck to each of its ends. The length of the entire capsule is 2 cm. The capacity of the capsule is

(A) 0.33 cm^3 (B) 0.34 cm^3

(C) 0.35 cm^3 (D) 0.36 cm^3

Q.62 The length of the longest pole that can be kept in a room (12 m × 9 m × 8 m) is

(A) 29 m (B) 21 m

(C) 19 m (D) 17 m

Q.63 The length of the diagonal of a cube is $6\sqrt{3}$ cm. Its total surface area is
(A) 144 cm^2 (B) 216 cm^2
(C) 180 cm^2 (D) 108 cm^2

Q.64 The volume of a cube is 2744 cm^3. Its surface area is
(A) 196 cm^2 (B) 1176 cm^2
(C) 784 cm^2 (D) 588 cm^2

Q.65 The total surface area of a cube is 864 cm^2. Its volume is
(A) 3456 cm^3 (B) 432 cm^3
(C) 1728 cm^3 (D) 3456 cm^3

Q.66 How many bricks each measuring (25 cm × 11.25 cm × 6 cm) will be required to construct a wall (8 m × 6 m × 22.5 m) ?
(A) 8000 (B) 6400
(C) 4800 (D) 7200

Q.67 The area of the base of a rectangular tank is 6500 cm^2 and the volume of water contained in it is 2.6 m^3. The depth of water in the tank is
(A) 3.5 m (B) 4 m
(C) 5 m (D) 8 m

Q.68 The volume of a wall, 5 times as high as it is broad and 8 times as long as it is high, is 12.8 m^3. The breadth of the wall is
(A) 30 cm (B) 40 cm
(C) 22.5 cm (D) 25 cm

Q.69 If the areas of three adjacent faces of a cuboid are x, y, z respectively, then the volume of the cuboid is
(A) xyz (B) 2xyz
(C) $\sqrt{xyz}$ (D) $3\sqrt{xyz}$

Q.70 The sum of length breadth and height of a cuboid is 19 cm and its diagonal is $5\sqrt{5}$ cm. Its surface area is
(A) 361 cm^2 (B) 125 cm^2
(C) 236 cm^2 (D) 486 cm^2

Q.71 If each edge of a cube is increased by 50%, the percentage increase in the surface area is
(A) 50% (B) 75%
(C) 100% (D) 125%

Q.72 How many bags of grain can be stored in a cuboidal granary (8 m × 6 m × 3 m), if each bag occupies a space of 0.64 m^3 ?
(A) 8256 (B) 90
(C) 212 (D) 225

Q.73 A Cube of side 6 cm is cut into a number of cubes each of side 2 cm. The number of cubes formed is
(A) 6 (B) 9
(C) 12 (D) 27

Q.74 In a shower, 5 cm of rain falls. The volume of the water that falls on 2 hectares of ground is
(A) 100 m^3 (B) 10 m^3
(C) 1000 m^3 (D) 10000 m^3

Q.75 Two cubes have their volumes in the ratio 1 : 27. The ratio of their surface areas is
(A) 1 : 3 (B) 1 : 8
(C) 1 : 9 (D) 1 : 18

Q.76 The diameter of the base of a cylidner is 4 cm and its height is 14 cm. The volume of the cylinder is
(A) 176 cm^3 (B) 196 cm^3
(C) 276 cm^3 (D) 352 cm^3

Q.77 The diameter of a cylinder is 28 cm and its height is 20 cm. The total surface area of the cylinder is
(A) 2993 cm^2 (B) 2992 cm^2
(C) 2292 cm^2 (D) 2229 cm^2

Q.78 The height of a cylinder is 14 cm and its curved surface area is 264 cm^2. The volume of the cylinder is
(A) 308 cm^3 (B) 396 cm^3
(C) 1232 cm^3 (D) 1848 cm^3

Q.79 The curved surface area of a cylinder is 1760 cm^2 and its base radius is 14 cm. The height of the cylinder is
(A) 10 cm (B) 15 cm
(C) 20 cm (D) 40 cm

Q.80 The ratio of the total surface area to the lateral surface area of a cylinder with base radius 80 cm and height 20 cm, is
(A) 2 : 1 (B) 3 : 1
(C) 4 : 1 (D) 5 : 1

Q.81 The curved surface area of a cylindrical pillar is 264 m^2 and its volume is 924 m^3. The height of the pillar is
(A) 4 m (B) 5 m
(C) 6 m (D) 7 m

Q.82 The ratio between the radius of the base and the height of the cylinder is 2 : 3. If its volume is 1617 cm^3, the total surface area of the cylinder is
(A) 308 cm^2 (B) 462 cm^2
(C) 540 cm^2 (D) 770 cm^2

Q.83 The radii of two cylinder are in the ratio 2 : 3 and their heights are in the ratio 5 : 3. The ratio of their volumes is
(A) 27 : 20 (B) 20 : 27
(C) 4 : 9 (D) 9 : 4

Q.84 Two circular cylinders of equal volume have their height in the ratio 1 : 2. The ratio of their radii is
(A) $1 : \sqrt{2}$ (B) $\sqrt{2} : 1$
(C) 1 : 2 (D) 1 : 4

Q.85 The radius of the base of a cone is 5 cm and its height is 12 cm, its curved surface area is
(A) 60 π cm^2 (B) 65 π cm^2
(C) 30 π cm^2 (D) none of these

Q.86 The diameter of the base of a cone is 42 cm and its volume is 12936 cm^3. Its height is
(A) 28 cm (B) 21 cm
(C) 35 cm (D) 14 cm

Q.87 The area of the base of a right circular cone is 154 cm^2 and its height is 14 cm. Its curved surface area is
(A) $154\sqrt{5}$ cm^2 (B) $154\sqrt{7}$ cm^2
(C) $77\sqrt{7}$ cm^2 (D) $77\sqrt{5}$ cm^2

Q.88 On increasing each of the radius of the base and the height of a cone by 20% its volume will be increased by
(A) 20% (B) 40%
(C) 60% (D) 72.8%

Q.89 The radii of the base of a cylinder and a cone are in the ratio between their volumes is
(A) 9 : 8 (B) 3 : 4
(C) 8 : 9 (D) 4 : 3

Q.90 A metallic cylinder of radius 8 cm and height 2 cm is melted and converted into a right circular cone of height 6 cm. The radius of the base of this cone is
(A) 4 cm (B) 5 cm
(C) 6 cm (D) 8 cm

Q.91 The height of a conical tent is 14 m and its floor area is 346.5 m^2. How much canvas, 1.1 wide, will be required for it ?
(A) 490 m (B) 525 m
(C) 665 m (D) 860 m

Q.92 The diameter of a sphere is 14 cm. Its volume is
(A) 1428 cm^3 (B) 1439 cm^3
(C) $1437\frac{1}{3}$ cm^3 (D) 1440 cm^3

Q.93 The ratio between the volumes of two spheres is 8 : 27. What is the ratio between their surface areas
(A) 2 : 3 (B) 4 : 5
(C) 5 : 6 (D) 4 : 9

Q.94 A hollow metallic sphere with external diameter 8 cm and internal diameter 4 cm is melted and moulded into a cone having base radius 8 cm. The height of the cone is
(A) 12 cm (B) 14 cm
(C) 15 cm (D) 18 cm

Q.95 A metallic cone having base radius 2.1 cm and height 8.4 cm is melted and moulded into a sphere. The radius of the sphere is **[NTSE]**
(A) 2.1 cm (B) 1.05 cm
(C) 1.5 cm (D) 2 cm

Q.96 The area of a rhombus is 120cm^2. If one of the diagonals is of length 10cm, then the length of one of its sides is- **[NTSE]**
(A) 12cm (B) 13cm
(C) 24cm (D) $2\sqrt{30}$m

Q.97 A square and an equilateral triangle are inscribed in a circle. If a and b denote the lengths of their sides, then **[NTSE]**
(A) $a^2 = \frac{b^2}{2}$ (B) $\frac{a^2}{2} = b^2$
(C) $3b^2 = 2a^2$ (D) $3a^2 = 2b^2$

Q.98 One diagonal of a parallelogram is 70cm and the perpendicular distance of this diagonal from either of the outlying vertices is 27cm. The area of parallelogram (in cm^2) is **[NTSE]**
(A) 1800 (B) 1836
(C) 1890 (D) 1990

Q.99 The perimeter of a triangle is 100cm and its sides are in the ratio 1 : 2 : 3. The area of the triangle (in cm^2) is **[NTSE]**

(A) $100\sqrt{3}$ (B) $100\sqrt{15}$

(C) $100\sqrt{5}$ (D) $100\sqrt{7}$

Q.100 In a circle of radius 7cm, arc subtends an angle of 108° at the centre. The area of the sector is $\left(\pi = \frac{22}{7}\right)$ **[NTSE]**

(A) 43.2 cm^2 (B) 44.2 cm^2

(C) 45.2 cm^2 (D) 46.2 cm^2

Q.101 The sum of areas of two circles A and B is equal to the area of a third circle C whose diameter is 30cm. If the diameter of circle A is B cm, then the radius of circle B is **[NTSE]**

(A) 18cm (B) 15cm

(C) 12cm (D) 10cm

Q.102 The area of a trapezium is 275cm^2. If the parallel sides are in the ratio 2 : 3 and the perpendicular distance between them is 5cm, then the smaller of the parallel sides is **[NTSE]**

(A) 36cm (B) 40cm

(C) 44cm (D) 48cm

Q.103 The volume of a cube is 343 cubic cm. The surface area of the cube is **[NTSE]**

(A) 284 cm^2 (B) 288 cm^2

(C) 290 cm^2 (D) 294 cm^2

Q.104 Three cubes of metal whose edges are 3cm, 4cm and 5cm, respectively are melted and a new cube is formed. The diagonal of the new cube is **[NTSE]**

(A) $4\sqrt{3}$ cm (B) 6cm

(C) $6\sqrt{3}$ cm (D) 8cm

Q.105 A solid sphere of radius r is sliced by the planes passing through its centre and perpendicular to each other. The total surface area of each of the pieces so formed is **[NTSE]**

(A) $\frac{2}{3}\pi r^2$ (B) πr^2

(C) $\frac{4}{3}\pi r^2$ (D) $2\pi r^2$

ANSWER KEY

1.	B	**2.**	A	**3.**	C	**4.**	D
5.	D	**6.**	B	**7.**	C	**8.**	D
9.	B	**10.**	A	**11.**	D	**12.**	A
13.	B	**14.**	C	**15.**	B	**16.**	C
17.	B	**18.**	C	**19.**	A	**20.**	D
21.	D	**22.**	B	**23.**	A	**24.**	C
25.	D	**26.**	B	**27.**	A	**28.**	B
29.	C	**30.**	B	**31.**	C	**32.**	C
33.	A	**34.**	D	**35.**	A	**36.**	B
37.	C	**38.**	A	**39.**	B	**40.**	D
41.	B	**42.**	A	**43.**	B	**44.**	C
45.	C	**46.**	B	**47.**	C	**48.**	A
49.	D	**50.**	C	**51.**	C	**52.**	B
53.	C	**54.**	C	**55.**	A	**56.**	B
57.	D	**58.**	B	**59.**	A	**60.**	B
61.	D	**62.**	D	**63.**	B	**64.**	B
65.	C	**66.**	B	**67.**	B	**68.**	B
69.	C	**70.**	C	**71.**	D	**72.**	D
73.	D	**74.**	C	**75.**	C	**76.**	A
77.	B	**78.**	B	**79.**	C	**80.**	D
81.	C	**82.**	D	**83.**	B	**84.**	B
85.	B	**86.**	A	**87.**	A	**88.**	D
89.	A	**90.**	D	**91.**	B	**92.**	C
93.	D	**94.**	B	**95.**	A	**96.**	B
97.	D	**98.**	C	**99.**	B	**100.**	D
101.	C	**102.**	C	**103.**	D	**104.**	C
105.	D						